Math 3280 Assignment 3. Due Friday, September 16th.

In addition to the problems below you should read sections 1.6, 2.4, 2.5, and 2.6 in the text. We will not cover the material on the Bernoulli substitution in 1.6 however.

Solve the following linear initial value problems.

(1) $xy' = y + 2x, \;\; y(1) = 2.$

(2) $y' + 4y = 2xe^{-4x}, \;\; y(0) = 0.$

(3) $y' = \cos(x) - y\cos(x), \;\; y(0) = 1.$

(4) $y' = 1 + 2xy, \;\; y(0) = 1.$ Your answer can be written in terms of the error function, $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$

(5) Consider a tank containing 1000 liters (L) of brine with 100 kilograms (kg) of salt dissolved. Pure water is added to the tank at a rate of 10 L/s, and stirred mixture is drained out at a rate of 10 L/s. Find the time at which only 1 kg of salt is left in the tank.

(6) Consider two tanks, with the first tank draining into the second. The first tank has 10 liters of a solution containing 200 grams of a dye dissolved in it. It drains into the second tank at a rate of 1 L/s, while being refilled with pure water at the same rate. The second tank initially contains 100 liters of pure water and is being emptied at a rate of 1 L/s. Both tanks are well-stirred at all times. Find the maximum concentration of dye in the second tank.

(7) Use the substitution $v = y/x$ to solve the IVP: $y' = \frac{2xy}{x^2 - y^2}$, $y(0) = 2$.

(8) Use Euler's method and the 4th-order Runge-Kutta method to estimate $x(1)$ if $x(0) = 1$ and $\frac{dx}{dt} = x + t^2$, using 2 steps. For this question you can use a calculator but you should write out the steps explicitly.