Math 3280 Assignment 5, due October 3rd.

(1) A linear system of the form:

$$a_{11}x + a_{12}y = 0$$

$$a_{21}x + a_{22}y = 0$$

is said to be homogeneous (i.e. when the right hand side is zero). Explain in geometric terms why such a system must either have a unique solution or infinitely many solutions. Find a formula for the unique solution when it exists.

(2) Use elementary row operations to transform the augmented coefficient matrix of the system below into echelon or reduced echelon form. Use this to write down all the solutions to the system.

$$\begin{aligned} x + 2y + -2z &= 3 \\ 2x + 4y - 2z &= 2 \\ x + 3y + 2z &= -1 \end{aligned}$$

(3) Same as above, for the system:

$$\begin{aligned} x + 2y - z &= -1 \\ x + 3y - 2z &= 0 \\ -x - z &= 3 \end{aligned}$$

(4) Under what conditions on the numbers a, b, and c does the following system have a unique solution, infinitely many solutions, or no solutions?

$$\begin{aligned} 2x - y + 3z &= a \\ x + 2y + z &= b \\ 6x + 2y + 8z &= c \end{aligned}$$

(5) Reduce the following matrix to row reduced echelon form.

$$\begin{pmatrix} 1 & 3 & -6 \\ 2 & 6 & 7 \\ 3 & 9 & 1 \end{pmatrix}$$

For the following three pairs of matrices, compute whatever products between them are defined.

(6)

$$A = \begin{pmatrix} 1 & -2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

(7)

$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

(8)

$$A = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ -1 & 1 \end{pmatrix}$$

(9) Find a two by two matrix A with all non-zero entries such that $A^2 = 0$.

(10) Find a two by two matrix A with each diagonal element equal to zero such that $A^2 = -I$. (The existence of such matrices is important, as it lets us represent complex numbers by real matrices).

(11) Find the inverse A^{-1} of the following matrix by augmenting with the identity and computing the reduced echelon form.

$$A = \begin{pmatrix} 5 & 3 & 6 \\ 4 & 2 & 3 \\ 3 & 2 & 5 \end{pmatrix}$$

(12) Compute the inverse of A to find a matrix X such that $AX = B$ if $A = \begin{pmatrix} 5 & 3 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 6 & -4 \end{pmatrix}$.

(13) Show that if A, B, and C are invertible matrices then $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

The next two problems are much easier if you use some of the properties of determinants.

(14) Compute the determinant of $A = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$.

(15) Compute the determinant of $A = \begin{pmatrix} 3 & 1 & 0 & 2 \\ 0 & 1 & 20 & 2 \\ 0 & 0 & 1 & -63 \\ 9 & 3 & 0 & 1 \end{pmatrix}$.

(16) Show that if a matrix A has the property $A^2 = A$, then either $det(A) = 0$ or $det(A) = 1$.

(17) A matrix is called orthogonal if $A^T = A^{-1}$. Show that if A is orthogonal then $det(A) = \pm 1$.