

Math 3280 Assignment 7, due Friday October 21st.

For this assignment you should read sections 5.1, 5.2, and 5.3 in the text.

- (1) Compute the Wronskian of $f_1 = e^{-x}$, $f_2 = \cos(x)$ and $f_3 = \sin(x)$ to determine whether these three functions are linearly independent on the real line.
- (2) Solve the initial value problem $y'' - 4y = 0$, $y(0) = 4$, $y'(0) = 2$ given that $y_1 = e^{2x}$ and $y_2 = e^{-2x}$ are both solutions to the ODE.
- (3) Find the general solution to $y'' + 6y' = 0$.
- (4) Find the general solution to $4y'' + 4y' + y = 0$.
- (5) For what second-order constant coefficient linear homogeneous ODE would $y = C_1 + C_2x$ be the general solution?
- (6) Show that the functions $3x$, $2x^2$, and $5x - 8x^2$ are linearly dependent by finding a linear combination of them that equals zero.
- (7) Find the general solution to $y'' + 10y' + 25y = 0$.
- (8) Find the general solution to $y^{(4)} - 6y^{(3)} + 9y'' = 0$.
- (9) Solve the initial value problem $y'' - 6y' + 25y = 0$, $y(0) = 6$, $y'(0) = 2$.
- (10) Find the general solution of $6y^{(4)} + 5y^{(3)} + 18y'' + 20y' - 24y = 0$ given that $y = \cos(2x)$ is a solution.
- (11) Consider the differential equation $y'' + \operatorname{sgn}(x)y = 0$, where $\operatorname{sgn}(x)$ is the sign function:

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Compute the two linearly independent solutions y_1 and y_2 of this differential equation which satisfy the initial conditions $y_1(0) = 1$, $y_1'(0) = 0$ and $y_2(0) = 0$, $y_2'(0) = 1$. (First solve the differential equation for $x < 0$ and $x > 0$, and then use the initial conditions to glue them together.)