Math 3280 Assignment 2, due Friday September 8th.

The problems below are primarily based on sections 1.3, 1.4, and 1.5 in our text, which I recommend you read.

Find the general solutions $y(x)$ to the following separable equations:

(1) $y' = 4xy$.

(2) $(2 + 2x)y' = 4y$.

(3) $y' = y\cos(x)$.

(4) $y' = 1 + x + y + xy$.

Find the solution $y(x)$ to the following initial value problems:

(5) $y' = 2ye^x$, $y(0) = 2e^2$.

(6) $y' = x^3(y^2 + 1)$, $y(0) = 1$.

(7) In carbon-dating organic material it is assumed that the amount of carbon-14 (^{14}C) decays exponentially ($\frac{d\ ^{14}C}{dt} = -k\ ^{14}C$) with rate constant of $k \approx 0.0001216$ where t is measured in years. Suppose an archeological bone sample contains 1/7 as much carbon-14 as is in a present-day sample. How old is the bone?

(8) Suppose you are designing a dosage regimen for the antibiotic ciprofloxacin (cipro). Assume the half-life of cipro is 4 hours. Let $x(t)$ be the amount of cipro present at time t hours after the initial dose, in units of milligrams per kilogram of the patient, and that x satisfies the differential equation $x' = -kx$ where k is a constant. If you decide to use equal doses which increase the value of x by 10 every 4 hours, how large can x become over time?

Determine what the existence and uniqueness theorem (Theorem 1 from Chapter 1.3) guarantees about solutions to the following initial value problems (note that you do not have to find the solutions):

(9) $dy/dx = \sqrt{xy}$, $y(0) = 1$.

(10) $dy/dx = y^{1/3}$, $y(0) = 2$.

(11) $dy/dx = y^{1/3}$, $y(2) = 0$.

(12) $dy/dx = x\ln(y)$, $y(0) = 1$.

Solve the following first-order linear ODEs:

(13) $dy/dx = -2y + 2xe^{-2x}$.

(14) $dy/dx + y\tan(x) = \sin(x)$.