Math 3280 Assignment 3. Due Friday, 9/15.

Solve the following linear initial value problems.

(1) $xy' = y + 2x, \;\; y(1) = 2.$

(2) $y' + 4y = 2xe^{-4x}, \;\; y(0) = 0.$

(3) $y' = \cos(x) - y\cos(x), \;\; y(0) = 1.$

(4) $y' = 1 + 2xy, \;\; y(0) = 1.$ Your answer can be written in terms of the error function, $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$

(5) Consider a tank containing 1000 liters (L) of brine with 100 kilograms (kg) of salt dissolved. Pure water is added to the tank at a rate of 10 L/s, and stirred mixture is drained out at a rate of 10 L/s. Find the time at which only 1 kg of salt is left in the tank.

(6) Consider two tanks, with the first tank draining into the second. The first tank has 10 liters of a solution containing 200 grams of a dye dissolved in it. It drains into the second tank at a rate of 1 L/s, while being refilled with pure water at the same rate. The second tank initially contains 100 liters of pure water and is being emptied at a rate of 1 L/s. Both tanks are well-stirred at all times. Find the maximum concentration of dye in the second tank.

(7) Use (a) Euler's method and (b) the 4th-order Runge-Kutta method to estimate $x(1)$ if $x(0) = 1$ and $\frac{dx}{dt} = x + t^2$, using 2 steps. For this question you can use a calculator but you should write out the steps explicitly.