Math 3280 Assignment 4, due Friday, September 22nd.

This material is mostly from Chapter 2.

(1) Suppose a population of invasive gophers, P, grows at a rate proportional to the square root of the population[1] . Initially the population size is 400 and it is increasing (initially) at a rate of 40 gophers/month. What will the population be after 1 year?

(2) Suppose a population of 15000 people are susceptible to a contagious disease, and that this disease spreads at a rate that is proportional to the number of infected people times the number of uninfected people. If there are initially 1000 people with the disease, and the number of infections is increasing at a rate of 140/day, how much longer will it take for half the population to be infected?

For the following three differential equations find the equilibria and determine their stability. Then solve the differential equation and use all of this information to sketch some of the typical trajectories.

(3) $y' = y - 5$

(4) $y' = y^2 - 3y$

(5) $y' = (y - 1)^2$

(6) Consider a fish population that would change according to the logistic equation, $P' = kP(M - P)$ if it were undisturbed. Suppose that these fish will be removed at a rate hP for some $h \geq 0$. If $k = 1$ and $M = 1000$, find the value of h that will maximize the number of removed fish at a stable equilibrium population.

(7) Use all three of the numerical methods (Euler, Improved Euler, and 4th-order Runge-Kutta) to find the value of $y(1)$ to four digits of accuracy using the smallest possible number of steps if $y' = xy - y^3$ and $y(0) = 1$. How many steps are necessary for each method? For this question you can use the functions from the lab 3 to do the computations.

(8) You jump out of an airplane at a height of 3000 meters. After 20 seconds, you open your parachute. Assume a linear air resistance with a drag acceleration of rv where $r = 0.15$ without that parachute and $r = 1.5$ with the parachute. How long does it take to reach the ground?

(9) Suppose that the drag acceleration was exactly proportional to the square of the velocity, so $\frac{dv}{dt} = -kv^2$. If there were no other forces involved, and the intial velocity and position were $v(0) = v_0$, $x(0) = x_0$, how far would a particle travel before coming to rest?

[1] This dependence is sometimes used for a geographically expanding population, in which there is an internal population plateaued at its maximum sustainable level surrounded by a thin zone of colonization. The size of the thin zone is proportional to the square root of the internal area.