Math 3280 Assignment 8, due Monday, November 6th.

(1) Find the general solution to $y^{(4)} - 6y^{(3)} + 9y'' = 0$.

(2) Find the general solution of $6y^{(4)} + 5y^{(3)} + 18y'' + 20y' - 24y = 0$ given that $y = \cos(2x)$ is a solution.

(3) Find a particular solution to the ODE $y'' - y' + 2y = 4x + 12$.

(4) Find a particular solution to the ODE $y'' - y' + y = \sin^2(x)$. (Hint: it may be helpful to use a trig identity.)

(5) Find the general solution to $y^{(3)} - y' = e^x$.

For the following two problems (6 and 7), determine the form of the particular solution - note that **you do not have to determine the values of the coefficients**. You should not include terms from the homogeneous (complementary) solution.

(6) Determine the form of the particular solution to $y''' = 9x^2 + 1$.

(7) Determine the form of the particular solution to $y^{(4)} - 16y'' = x^2 \sin(4x) + \sin(4x)$.

(8) Solve the initial value problem $y'' + 2y' + 2y = \cos(3x)$, $y(0) = 0$, $y'(0) = 2$.

(9) Solve the initial value problem $y^{(4)} - y = 1$, $y(0) = y'(0) = y''(0) = y^{(3)} = 0$.

(10) How many times can an overdamped mass-spring system ($mx'' + cx' + kx = 0$ with $c^2 > 4mk$; c, m, and k are non-negative) with arbitrary initial conditions $x(0) = x_0$, $x'(0) = v_0$ pass through $x = 0$? What if it is critically damped ($c^2 = 4mk$)?

(11) Find the steady-state solution of the forced, damped oscillator $x'' + x'/4 + 2x = 2\cos(wt)$ if $x(0) = 0$ and $x'(0) = 4$. Sketch the overall amplitude of the steady-state solution as a function of w.

(12) Rewrite the second-order differential equation $x'' + 3x' + 5x = t$ as a system of first-order differential equations. (You do not have to find the solution.)

(13) Compute the Wronskian of $f_1 = e^{-x}$, $f_2 = \cos(x)$ and $f_3 = \sin(x)$ to determine whether these three functions are linearly independent on the real line.

Find the eigenvalues and eigenvectors of the following matrices:

(14) $\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$

(15) $\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$

(16) $\begin{pmatrix} 2 & 0 & 0 \\ 5 & 3 & -2 \\ 2 & 0 & 1 \end{pmatrix}$

(17) $\begin{pmatrix} 3 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(18) $\begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}$

(19) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$