Math 3280 Assignment 9, due Wednesday, November 15th.

Find a matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix, for the following matrices if such a P exists.

(1) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$

(2) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

(3) Show that if A is invertible and λ is an eigenvalue of A, then $1/\lambda$ is an eigenvalue of A^{-1}. Are the eigenvectors the same?

(4) By computing the eigenvalues and eigenvectors of $A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$ find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix. Use this diagonalization to compute A^6.

(5) Find the general solution to the system $x_1' = x_1 + 2x_2$, $x_2' = 2x_1 + x_2$. Sketch some of the solutions near the origin, including some that start on the lines spanned by the eigenvectors of the coefficient matrix of the system.

(6) Find the general solution to the system $x_1' = x_1 + 2x_2$, $x_2' = 3x_1 + 2x_2$.

(7) Find the general solution to the system $x_1' = x_1 - 5x_2$, $x_2' = x_1 - x_2$. Sketch some of the solutions near the origin.

(8) Solve the initial value problem $x_1' = x_1 + 2x_2$, $x_2' = -2x_1 + x_2$, $x_1(0) = 1$, $x_2(0) = 0$.