

Math 3280 Worksheet 10: bifurcations of autonomous ODEs

Group members (2 to 4): \_\_\_\_\_

- (1) Suppose that before fishing is allowed, a population of fish in a lake satisfies the differential equation  $\frac{dP}{dt} = P(1000 - P)$  where  $t$  is measured in years. Then fishing is allowed at a rate proportional to the population, so that  $\frac{dP}{dt}$  is decreased by  $hP$  where  $h$  is a parameter.
- (a) Sketch a bifurcation diagram in the  $(h, P)$  plane that shows the equilibria of the system as  $h$  is varied and their stability.

(b) If  $h = 500$ , and the lake begins in an overstocked state with  $P = 2000$ , what value will the population approach over time?

(c) If  $h = 1000$ , and the lake begins in an overstocked state with  $P = 2000$ , what value will the population approach over time?