Math 3280 Worksheet 13: Reduced row echelon form; matrix multiplication

Group members (2 to 4):

Compute the rref (reduced row echelon form) of the following matrices in problems 1 and 2. The rref is defined by the properties:

- Any all-zero rows are below non-zero rows
- The first (leftmost) nonzero entry in each row, called a pivot, is equal to 1.
- Everything above and below a pivot (in the same column) is zero.
- Each pivot is to the right of the pivots above it.

$$\begin{pmatrix} 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$(1) \left( \begin{array}{rrrr} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & -2 & -3 \end{array} \right)$$

(2) 
$$\begin{pmatrix} a & a & b \\ b & b & c \\ 0 & 0 & -1 \end{pmatrix}$$
 where  $a, b, and c$  are distinct and nonzero.

(3) If 
$$B = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$
 and  $C = \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$ , compute both *BC* and *CB*.

(4) Optional, extra credit (worth 1/2 of a worksheet): Describe all two by two matrices  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  which commute with  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ .