Math 3280 Worksheet 15: Matrix inverses and elementary matrices.

Group members (2 to 4): ____

Don't overlook part 2 of this worksheet on the back!

(1) Find the inverse of the following matrix A by using row operations (multiplying rows, adding a multiple of one row to another, and interchanging rows) on the matrix A adjoined to the 3×3 identity matrix. Indicate at each step what row operations you are using. You should verify that your answer really is the inverse to A by multiplying it by A to obtain the identity.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

(2) Write the inverse from the previous problem as a product of elementary matrices by representing each of the row operations you used as elementary matrices. Here is an example. From the following row-reduction,

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{pmatrix} \xrightarrow{-2R_1+R_2} \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{pmatrix} \xrightarrow{-R_2} \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{pmatrix}$$

$$\xrightarrow{-R_2+R_1} \begin{pmatrix} 2 & 0 & -1 & 1 \\ 0 & 1 & 2 & -1 \end{pmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 2 & -1 \end{pmatrix}$$

we can write the inverse (elementary matrices ordered **right-to-left** instead of left-to-right):

$$\begin{pmatrix} -1/2 & 1/2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$