Math 3280 Worksheet 18: Curve-fitting with matrices

Group members (2 to 4): _____

(1) Using the equation

$$Ax + By + C = x^2 + y^2,$$

find a circle passing through the points $(1, 7)$, $(-1, 3)$, and $(0, 0)$. After finding A, B, and C, put the circle's equation in the standard form

$$(x - c_x)^2 + (y - c_y)^2 = r^2.$$

(by completing the squares).

(2) Find the ellipse of minimal area which is of the form

$$Ax^2 + 2Bxy + Cy^2 = 1$$

and which passes through the points $(x, y) = (1, 1)$ and $(x, y) = (-1, 2)$.

The area of these ellipses is most easily expressed by writing the equation as

$$(x, y)M \begin{pmatrix} x \\ y \end{pmatrix} = 1$$

where $M = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$. Then the area of such an ellipse is $\dfrac{\pi}{\sqrt{\det(M)}}$ (note that the minimal area will occur when $det(M)$ is maximized).