

Math 3280 Worksheet 39: Solving initial value problems with Laplace transforms

Group members (2 to 4): _____

- (1) Solve the initial value problem $x'' + x' - 2x = t$, $x(0) = x'(0) = 0$ using the Laplace transform. If you have time, check your work using other methods (characteristic equation and undetermined coefficients). A table of Laplace transforms is given on the back of this sheet.

| Function $f(t)$ | Transform $\mathcal{L}(f(t)) = F(s)$ |
|--|--------------------------------------|
| 1 | $\frac{1}{s}$ |
| t | $\frac{1}{s^2}$ |
| t^n (n is a non-negative integer) | $\frac{n!}{s^{n+1}}$ |
| t^a ($a > -1$) | $\frac{\Gamma(a+1)}{s^{a+1}}$ |
| e^{kt} | $\frac{1}{s-k}$ |
| $\cos(kt)$ | $\frac{s}{s^2+k^2}$ |
| $\sin(kt)$ | $\frac{k}{s^2+k^2}$ |
| $-tf(t)$ | $F'(s)$ |
| $e^{at}f(t)$ | $F(s-a)$ |
| $f'(t)$ | $sF(s) - f(0)$ |
| $f''(t)$ | $s^2F(s) - sf(0) - f'(0)$ |

TABLE 1. Some Laplace transforms, $\mathcal{L}(f(t)) = F(s)$