Math 3280 Worksheet 41: Solving systems of linear ODEs with Laplace transforms

Group members (2 to 4): _____

(1) Use the Laplace transform method to solve the initial value problem

$$x' = x + 2y,$$
$$y' = x + e^t,$$
$$x(0) = 0, \quad y(0) = 0.$$

(Note that once you find either $x(t)$ or $y(t)$, the other can be computed from the system of ODEs.)

Function	Transform
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n (n is a non-negative integer)	$\frac{n!}{s^{n+1}}$
t^a ($a > -1$)	$\frac{\Gamma(a+1)}{s^{a+1}}$
e^{kt}	$\frac{1}{s-k}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$e^{at}f(t)$	$F(s-a)$
$tx(t)$	$-X'(s)$
$\int_0^t x(\tau)d\tau$	$X(s)/s$
$x'(t)$	$sX(s) - x(0)$
$x''(t)$	$s^2X(s) - sx(0) - x'(0)$

TABLE 1. Some Laplace transforms, $\mathcal{L}(x(t)) = X(s)$