Worksheet 45: Numerical methods/IVP review.

Group members (2 to 4): _____

(a) Solve the initial value problem x'' + 2x' + 5x = t, x(0) = 0, x'(0) = 0, and find the exact value of x(2).

(b) Rewrite the ODE as a first order system.

(c) Approximate the solution x(2) using two steps of the Euler method. Compare this with an approximation using one step of the Improved Euler Method, and to the exact value.

The Improved Euler method is:

$$k_1 = f(t_n, \vec{x}_n)$$
$$k_2 = f(t_n + h, \vec{x}_n + hk_1)$$

$$\vec{x}_{n+1} = \vec{x}_n + h(k_1 + k_2)/2$$

 $t_{n+1} = t_n + h$

For this worksheet, $\vec{x} = (x, v)$ where v = x'. Each slope k_i is also a two-component vector, and the function f is vector-valued (i.e. it outputs a two-component vector).