Math 3280 Worksheet 46, optional review, do not hand in.

(1) Consider an long cascade of tanks, each containing 1 liter of water. Each tank drains into the next at a rate of 1 liter per hour. Initially the first tank contains 1 gram of salt dissolved into it, but it is being refilled with pure water at a rate of 1 liter per hour. The other tanks in the cascade are initially filled with pure water. Compute how much salt is in the nth tank at time t.

(2) Compute the inverse of

$$A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(3) Are the vectors $v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$, $v_4 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

linearly independent? If not, write one of them as a linear combination of the others.