Math 3280 Worksheet 8: Improved Euler and Runge-Kutta methods for numerical solutions

Group members (2 to 4): _____

(1) For the initial value problem $y(1) = 1$ and $\frac{dy}{dx} = \left(\frac{y}{x}\right)\left(\frac{y-x}{x+y}\right)$, approximate $y(2)$ by using:

(a) 1 step of the improved Euler method

(b) 1 step of the 4th-order Runge-Kutta method (formulae on reverse).

(2) For extra credit try to find the most accurate answer you can; a bonus will be given for the answer with the most correct digits.

The 4th-order Runge-Kutta formulae for the ODE $\frac{dy}{dx} = f(x, y)$, going from (x_i, y_i) to (x_{i+1}, y_{i+1}) with stepsize h are:

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + h/2, y_i + hk_1/2)$$
$$k_3 = f(x_i + h/2), y_i + hk_2/2)$$
$$k_4 = f(x_i + h, y_i + hk_3)$$

And then:

$$y_{i+1} = y_i + h(k_1 + 2k_2 + 2k_3 + k_4)/6$$
$$x_{i+1} = x_i + h$$