Math 3280 Assignment 13; due May 9th.

(1) Find the power series solution to the ODE $y' = 3x^2y$ (expanded at $x = 0$). You should be able to determine each coefficient as an explicit function of its index (rather than just a recurrence relation).

(2) Show that the coefficients of the power series solution to the initial value problem $y'' - y' - y = 0$, $y(0) = 0$, $y'(0) = 1$ have the form $c_n = F_n/n!$ where F_n is the nth Fibonacci number. (The Fibonacci numbers are $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$, satisfying the recursion relation that each number is the sum of the previous two in the sequence.)

(3) Determine the power series solution and radius of convergence of the ODE $y'' + x^2y = 0$.