

Math 3280 Assignment 6, due Friday March 14th.

- (1) Find a quadratic polynomial $a_2x^2 + a_1x + a_0$ whose graph passes through the points $(1, 3)$, $(2, 3)$, and $(4, 9)$.
- (2) Determine whether the vectors $(0, 2)$ and $(0, 5)$ are linearly dependent or independent.
- (3) Express $w = (1, 2)$ as a linear combination of $u = (-1, -1)$ and $v = (2, 1)$.
- (4) Calculate the determinant of the matrix whose columns are u , v , and w to determine if u , v , and w are linearly independent or not, with $u = (-2, -5, -4)$, $v = (5, 4, -6)$, and $w = (8, 3, -4)$.
- (5) Is the subset $W = \{(x, y, z) \mid y \geq 0\} \subset \mathbb{R}^3$ a vector subspace of \mathbb{R}^3 ? Explain why or why not.
- (6) Is the subset $W = \{(x, y, z) \mid y = 0\} \subset \mathbb{R}^3$ a vector subspace of \mathbb{R}^3 ? Explain why or why not.
- (7) Is the subset $W = \{(x, y, z) \mid z = 1\} \subset \mathbb{R}^3$ a vector subspace of \mathbb{R}^3 ? Explain why or why not.
- (8) If W is the subset of all vectors (x, y) in \mathbb{R}^2 such that $|x| = |y|$, is W a vector subspace or not?
- (9) Suppose that x_0 is a solution to the equation $Ax = b$ (where A is a matrix and x and b are vectors). Show that x is a solution to $Ax = b$ if and only if $y = x - x_0$ is a solution to the system $Ay = 0$.
- (10) Determine whether the vectors $v_1 = (3, 0, 1, 2)$, $v_2 = (1, -1, 0, 1)$, and $v_3 = (4, 2, 2, 2)$ are linearly independent or dependent. If they are linearly dependent, find a non-trivial combination of them that adds up to the zero vector.
- (11) Find a basis for the subspace of \mathbb{R}^3 given by $x - 2y + 7z = 0$.
- (12) Find a basis for the subspace of \mathbb{R}^3 given by $x = z$.
- (13) Find a basis for the subspace of all vectors (x_1, x_2, x_3, x_4) in \mathbb{R}^4 such that $x_1 + x_2 = x_3 + x_4$.

The following two questions are about subsets of the set of real-valued functions of the real line. We will call this set \mathcal{F} .

- (14) Is the subset of \mathcal{F} with the property that $f(0) = 0$ a vector space?
- (15) Is the subset of \mathcal{F} with the property that $f(-x) = -f(x)$ for all x a vector space?
- (16) Compute the Wronskian of $f_1 = e^{-x}$, $f_2 = \cos(x)$ and $f_3 = \sin(x)$ to determine whether these three functions are linearly independent on the real line.