Math 3280 Assignment 7, due Thursday, March 27th.

For this assignment you should read sections 5.1, 5.2, and 5.3 in the text.

(1) Solve the initial value problem $y'' - 4y = 0$, $y(0) = 4$, $y'(0) = 2$ given that $y_1 = e^{2x}$ and $y_2 = e^{-2x}$ are both solutions to the ODE.

(2) Find the general solution to $y'' + 6y' = 0$.

(3) Find the general solution to $4y'' + 4y' + y = 0$.

(4) For what second-order constant coefficient linear homogeneous ODE would $y = C_1 + C_2 x$ be the general solution?

(5) Show that the functions $3x$, $2x^2$, and $5x - 8x^2$ are linearly dependent by finding a linear combination of them that equals zero.

(6) Find the general solution to $y'' + 10y' + 25y = 0$.

(7) Find the general solution to $y^{(4)} - 6y^{(3)} + 9y'' = 0$.

(8) Solve the initial value problem $y'' - 6y' + 25y = 0$, $y(0) = 6$, $y'(0) = 2$.

(9) Find the general solution of $6y^{(4)} + 5y^{(3)} + 18y'' + 20y' - 24y = 0$ given that $y = \cos(2x)$ is a solution.

(10) Consider the differential equation $y'' + sgn(x)y = 0$, where $sgn(x)$ is the sign function:

$$sgn(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Compute the two linearly independent solutions y_1 and y_2 of this differential equation which satisfy the initial conditions $y_1(0) = 1$, $y_1'(0) = 0$ and $y_2(0) = 0$, $y_2'(0) = 1$. (First solve the differential equation for $x < 0$ and $x > 0$, and then use the initial conditions to glue them together.)