Math 3280 Assignment 9, due Friday, April 11th.

(1) Find the steady-state solution of the forced, damped oscillator $x'' + x'/4 + 2x = 2\cos(wt)$ if $x(0) = 0$ and $x'(0) = 4$. Sketch the overall amplitude of the steady-state solution as a function of w.

(2) Rewrite the second-order differential equation $x'' + 3x' + 5x = t$ as a system of first-order differential equations. (You do not have to find the solution.)

Find the eigenvalues and eigenvectors of the following matrices:

(3) $\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$

(4) $\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$

(5) $\begin{pmatrix} 2 & 0 & 0 \\ 5 & 3 & -2 \\ 2 & 0 & 1 \end{pmatrix}$

(6) $\begin{pmatrix} 3 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(7) $\begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}$

(8) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Find a matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix, for the following matrices if such a P exists.

(9) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$

(10) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

(11) Show that if A is invertible and λ is an eigenvalue of A, then $1/\lambda$ is an eigenvalue of A^{-1}. Are the eigenvectors the same?

(12) By computing the eigenvalues and eigenvectors of $A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$ find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix. Use this diagonalization to compute A^6.