Math 3280 Worksheet 11: Linear air resistance

Group members (2 to 4): ____

(1) Suppose a baseball is hit at a height of 3 feet, at an angle $\theta = \arctan(4/3)$ relative to the ground. The initial velocity is 135 feet/second. If we ignore air resistance, the differential equations for the velocities of the ball in the horizontal (x) and vertical (y) directions are

$$\frac{dv_x}{dt} = 0 \qquad \frac{dv_y}{dt} = -g$$

where $g = 32$ feet/second^2, $v_x = \dfrac{dx}{dt}$ is the x-component of the velocity and $v_y = \dfrac{dy}{dt}$ is the y-component of the velocity. The initial conditions for these velocities are $v_x(0) = u\cos(\theta)$ and $v_y(0) = u\sin(\theta)$.

If we take $x(0) = 0$ and $y(0) = 3$, how far does the ball travel before hitting the ground (when $y = 0$)?

(2) Now use the following linear air resistance model for the baseball, which is much more realistic:

$$\frac{dv_x}{dt} = -rv_x \qquad \frac{dv_y}{dt} = -g - rv_y$$

Find $x(t)$ and $y(t)$ (in units of feet and seconds) if $r = 1/5$, $\theta = \arctan(4/3)$, and the initial velocity is 135 ft/s. (First find v_x and v_y, and then integrate with respect to t.) Then determine how far the ball travels before hitting the ground.

(3) Extra credit. Find the minimal velocity u and optimal angle θ for a home run in center field if the field wall is 10 feet high and 350 feet from home plate, assuming the air resistance model from part (1). (An exact answer may be impossible, so numerical answers are acceptable.) You must explain your reasoning. You can hand this in separately, individually if you prefer, by February 20th.