Group members (2 to 4): ________________________________

(1) Find a three by three matrix with strictly positive entries \((a_{ij} > 0\) for each entry \(a_{ij}\)) whose determinant is equal to 1. A bonus will be given for finding such a matrix with the smallest sum of entries, that is, such that \(\sum_{i=1,j=1}^{i=3,j=3} a_{ij} = a_{11} + a_{12} + \ldots + a_{33}\) is as low as possible.
(2) The $n \times n$ Vandermonde determinant is defined as:

$$V(x_1, x_2, \ldots, x_n) = \det \begin{pmatrix}
1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\
1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_n & x_n^2 & \cdots & x_n^{n-1}
\end{pmatrix}$$

Show that the $2 \times 2$ Vandermonde determinant $V(a, b) = b-a$. Then show that the $3 \times 3$ Vandermonde determinant $V(1, a, b)$ can be factored into $(a-1)(b-1)(b-a)$. 