

Math 3280 Worksheet 7: Improved Euler and Runge-Kutta methods for numerical solutions

Group members (2 to 4): _____

(1) For the initial value problem $y(1) = 1$ and $\frac{dy}{dx} = \left(\frac{y}{x}\right) \left(\frac{y-x}{x+y}\right)$, approximate

$y(2)$ by using:

(a) 2 steps of the improved Euler method

(b) 1 step of the 4th-order Runge-Kutta method (formulae on reverse).

- (2) For extra credit try to find the most accurate answer you can; a bonus will be given for the answer with the most correct digits.

The 4th-order Runge-Kutta formulae for the ODE $\frac{dy}{dx} = f(x, y)$, going from (x_i, y_i) to (x_{i+1}, y_{i+1}) with stepsize h are:

$$\begin{aligned}k_1 &= f(x_i, y_i) \\k_2 &= f(x_i + h/2, y_i + hk_1/2) \\k_3 &= f(x_i + h/2, y_i + hk_2/2) \\k_4 &= f(x_i + h, y_i + hk_3)\end{aligned}$$

And then:

$$\begin{aligned}y_{i+1} &= y_i + h(k_1 + 2k_2 + 2k_3 + k_4)/6 \\x_{i+1} &= x_i + h\end{aligned}$$