Math 3280 Assignment 12, due Wednesday, April 26th.
This assignment covers material from chapter 9.

(1) Compute the equilibria of the following nonlinear differential equations, and use that information to match each equation with a trajectory plot from the following page. It may be helpful to compute the eigenvalues at the equilibria.
  (a) $x' = x - y$, $y' = x + 3y - 4$.
  (b) $x' = 2x - y$, $y' = x - 3y$.
  (c) $x' = 2\sin(x) + \sin(y)$, $y' = \sin(x) + 2\sin(y)$.
  (d) $x' = x - 2y$, $y' = -x^3 + 4x$.
  (e) $x' = 1 - y^2$, $y' = x + 2y$.
  (f) $x' = x - 2y + 3$, $y' = x - y + 2$.

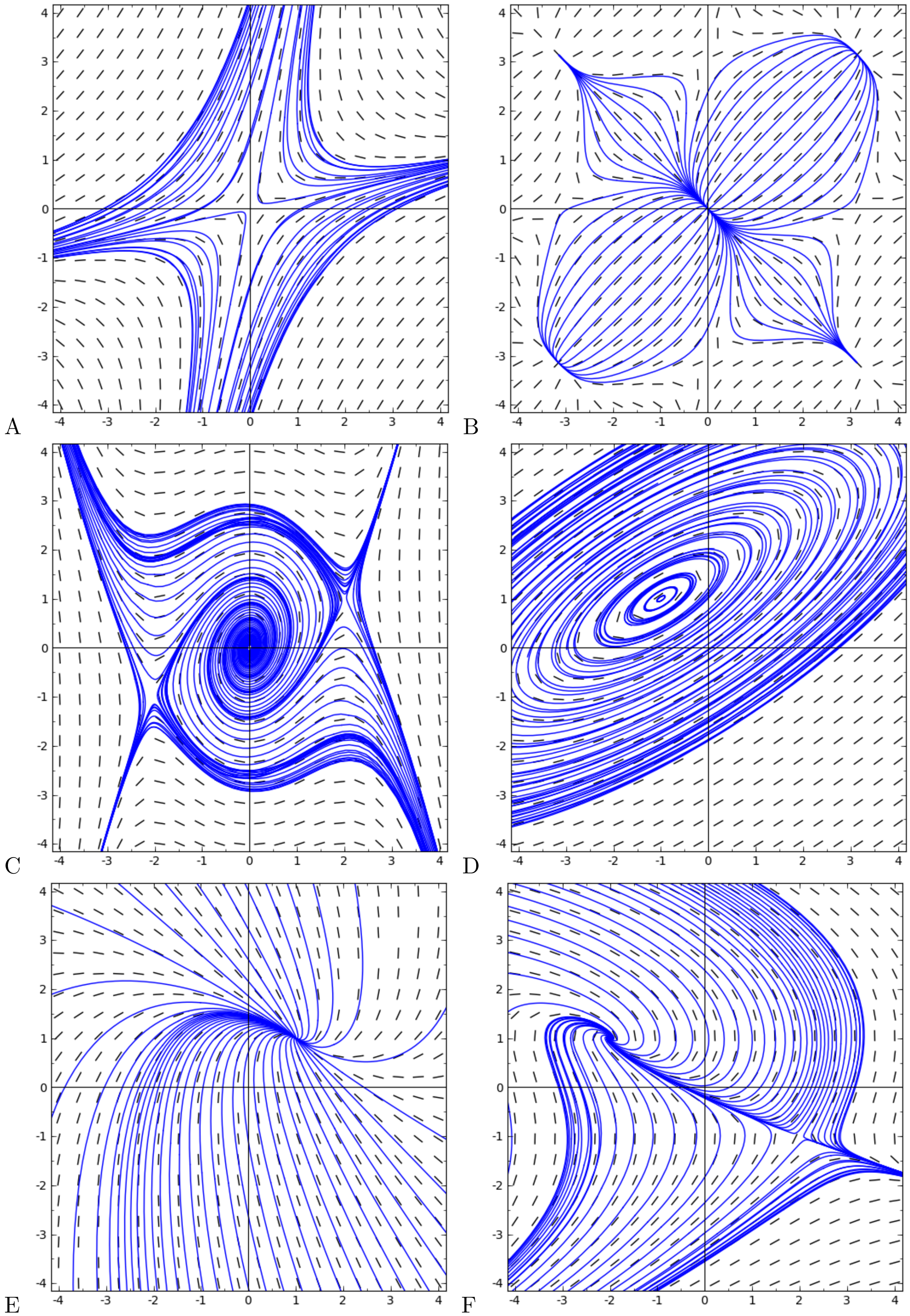
(2) Find the unique equilibrium of the system $x' = x - y$, $y' = 5x - 3y - 2$. Compute the eigenvalues of its linearization to determine the stability of the equilibrium (see Theorem 2 in section 9.2).

(3) Find the equilibria and the eigenvalues of their linearizations for the system $x' = 2x - y - x^2$, $y' = x - 2y + y^2$.

(4) In 1958, Tsuneji Rikitake formulated a simple model of the Earth's magnetic core to explain the oscillations in the polarity of the magnetic field. The equations for his model are:

$$\begin{aligned} x' &= -\mu x + yz \\ y' &= -\mu y + (z - a)x \\ z' &= 1 - xy \end{aligned}$$

where a and μ are positive constants. Find the equilibria for this system for $a = \mu = 1$, and determine the stability of the linearized system at those equilibria. It is OK to use a computer algebra system such as Sage to compute the eigenvalues of the linearized systems.


A
B
C
D
E
F