Math 3280 Worksheet 39: 2-dimensional linear systems

Group members (2 to 4): _____

(1) (a) Construct a two by two matrix A with zero trace and positive determinant (try to make it as simple as possible).

(b) What are its eigenvalues and eigenvectors?

(c) Sketch the solutions to the system $x' = Ax$ for your matrix (where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$).

(2) For the family of initial values problems $x_1(0) = 0$, $x_2(0) = 1$, and $x' = Ax$ where

$$A = \begin{pmatrix} 1 & 1 \\ 0 & a \end{pmatrix}$$

find the solution $x(t)$ assuming that $a > 0$, and then compute $\lim_{a \to 1} x$.