Worksheet 44: Numerical methods/IVP review.

Group members (2 to 4): _____

(a) Solve the initial value problem $x'' + 2x' + 5x = t$, $x(0) = 0$, $x'(0) = 0$, and find the exact value of $x(2)$.

(b) Rewrite the ODE as a first order system.

(c) Approximate the solution $x(2)$ using two steps of the Euler method. Compare this with two steps of the Improved Euler Method, and to the exact value.

The Improved Euler method is:

$$k_1 = f(t_n, \vec{x}_n)$$
$$k_2 = f(t_n + h, \vec{x}_n + hk_1)$$

$$\vec{x}_{n+1} = \vec{x}_n + h(k_1 + k_2)/2$$
$$t_{n+1} = t_n + h$$

For this worksheet, $\vec{x} = (x, v)$ where $v = x'$. Each slope k_i is also a two-component vector, and the function f is vector-valued (i.e. it outputs a two-component vector).