Worksheet 45 (ungraded, do not hand in)

Group members (2 to 4): _____

(1) Compute the inverse of

$$A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(2) Are the vectors $v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$, $v_4 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ linearly independent? If not, write one of them as a linear combination of the others.

(3) A matrix P is an *orthogonal projection* if $P^2 = P$ and $P^T = P$. Find the 3×3 orthogonal projection P that projects any 3-D vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ onto the line spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Hint: for this projection, $P \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $Pb = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ for any b that is perpendicular to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, such as $b = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.