Math 3280 Worksheet 46, do not hand in.

(1) Consider an long cascade of tanks, each containing 1 liter of water. Each tank drains into the next at a rate of 1 liter per hour. Initially the first tank contains 1 gram of salt dissolved into it, but it is being refilled with pure water at a rate of 1 liter per hour. The other tanks in the cascade are initially filled with pure water. Compute how much salt is in the nth tank at time t.

(2) The spread of many diseases are modeled by various SIR ODE models, where SIR is an acronym for Susceptible, Infected, and Recovered. In the following version, we assume a population has a constant proportional death rate of d and a birth rate of b. The disease is transmitted at a rate cIS, and infected people recover at a proportional rate I, giving the equations:

$$\frac{dS}{dt} = b - dS - cIS$$
$$\frac{dI}{dt} = cIS - (d+g)I$$
$$\frac{dR}{dt} = gI - dR$$

For a population with $b = d = 1$, when is the disease-free equilibrium point (disease free meaning $I = R = 0$) stable?