Math 3280 Assignment 1, due June 17th. This assignment is primarily on sections 1.1 - 1.5.

(1) Verify that the given function $y(x)$ is a solution to the differential equation by substitution for each of the problems below.

(a) $y' = 4x^3$, $y = x^4 + 27$.

(b) $y' = -3y$, $y = 2e^{-3x}$.

(2) Find all solutions of the form $y = e^{rx}$ to the differential equations below by substitution (here r is a real constant).

(a) $3y'' - 4y' - 4y = 0$.

(b) $4y'' = y$.

(3) Determine a value of the constant C so that the given solution of the differential equation satisfies the initial condition.

(a) $y = \ln(x + C)$ solves $e^y y' = 1, \quad y(0) = 1$.
(b) $y = Ce^{-x} + x - 1$ solves $y' = x - y, \quad y(0) = 3$.

(4) Write a differential equation for a population P that is changing in time (t) such that the rate of change is proportional to the square root of P.

(5) Solve the following initial value problems.

(a) $\frac{dy}{dx} = 3x + 1, \quad y(0) = 1$.

(b) $\frac{dy}{dx} = \sqrt{x}, \quad y(9) = 0$.

(c) $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}, \quad y(0) = 0$.

(d) $\frac{dy}{dx} = xe^{-x}, \quad y(0) = 2$.

(6) $y' = 4xy$.

(7) $(2 + 2x)y' = 4y$.

(8) $y' = y\cos(x)$.

(9) $y' = 1 + x + y + xy$.

Find the solution $y(x)$ to the following initial value problems:

(10) $y' = 2ye^x, \ y(0) = 2e^2$.

(11) $y' = x^3(y^2 + 1), \ y(0) = 1$.

(12) In carbon-dating organic material it is assumed that the amount of carbon-14 (^{14}C) decays exponentially $(\frac{d\,^{14}C}{dt} = -k\,^{14}C)$ with rate constant of $k \approx 0.0001216$ where t is measured in years. Suppose an archeological bone sample contains 1/7 as much carbon-14 as is in a present-day sample. How old is the bone?

(13) In this exercise you will work out a simplified alternate landing scenario for the Mars Science Laboratory spacecraft. You may wish to use a programmable calculator, Sage, or some other computational tool since the computations are unwieldy to do by hand.

Suppose the spacecraft is approaching Mars at a speed of $-470\ m/s$ at time $t = 0$, and at that time it is 25 km above the surface. We will ignore air resistance, so assume that the acceleration of the spacecraft is $-3.7278\ m/s^2$ (Mars surface gravitational acceleration).

(a) Write down the velocity and position of the spacecraft as functions of time.

(b) Now suppose that at time $t = t_f$ the spacecraft begins to fire rockets that change its total acceleration to $5\ m/s^2$. Compute the value of t_f such that the spacecraft lands on the surface with zero velocity. (Hint: it helps to introduce a time of landing, t_L, such that the height and velocity of the spacecraft are zero at $t = t_L$.)

Determine what the existence and uniqueness theorem (Theorem 1 from Chapter 1.3) guarantees about solutions to the following initial value problems (note that you do not have to find the solutions):

(14) $dy/dx = \sqrt{xy}$, $y(0) = 1$.

(15) $dy/dx = y^{1/3}$, $y(0) = 2$.

(16) $dy/dx = y^{1/3}$, $y(2) = 0$.

(17) $dy/dx = x \ln(y)$, $y(0) = 1$.

Solve the following first-order linear ODEs:

(18) $dy/dx = -2y + 2xe^{-2x}$.

(19) $dy/dx + y \tan(x) = \sin(x)$.