

Math 3280 Worksheet 27: Tank problem

(1) A tank is initially full of 2 liters of pure water. Then briny (salty) water is added at a rate of 2 liter/hour; however the concentration of brine in the incoming water varies as $c(t) = 1 - \sin(t)$ (grams/liter). Well-mixed fluid leaves the tank at the same rate (2 liter/hour).

(a) Write down the ODE and initial data for the amount of salt $y(t)$ in the tank for times $t \geq 0$.

(b) Solve the initial value problem. (There are at least three ways to do this: the equation for first-order linear ODEs using an integrating factor, the method of undetermined coefficients, and the Laplace transform.)

(c) Estimate the average concentration of salt in the tank as t becomes large. How would this be affected by increasing the volume of water initially in the tank?