

Math 3280 Worksheet 7: Equilibria of autonomous ODEs

Group members (2 to 4): _____

- (1) Sketch the phase plot (equilibria and direction of solutions) for the ODE $y' = y \sin y$ in the interval $y \in [-10, 10]$.

(2) Suppose that before fishing is allowed, a population of fish in a lake satisfies the differential equation $\frac{dP}{dt} = P(1000 - P)$ where t is measured in years. Then fishing is allowed at a rate proportional to the population, so that $\frac{dP}{dt}$ is decreased by hP where h is a parameter.

(a) Sketch a bifurcation diagram in the (h, P) plane that shows the equilibria of the system as h is varied.

(b) If $h = 500$, and the lake begins in an overstocked state with $P = 2000$, what value will the population approach over time?

(c) If $h = 1000$, and the lake begins in an overstocked state with $P = 2000$, what value will the population approach over time?