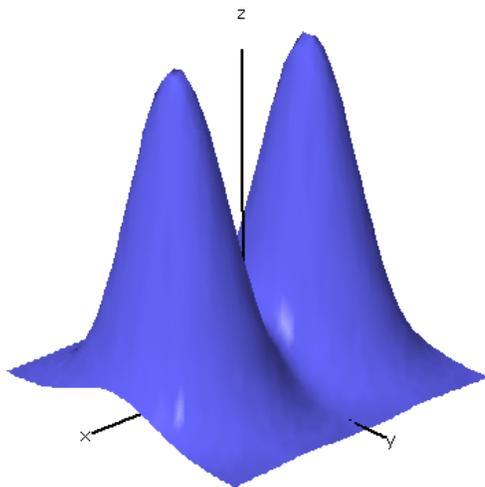


HOMEWORK 3, DUE WEDNESDAY SEPTEMBER 20TH.

This assignment is on material covered in sections 14.1 to 14.4 in Stewart's text. Reading (or at least skimming) those sections is highly recommended.

- (1) Sketch the contour map of the function whose graph is shown below.



- (2) Sketch a contour map of the function $f = x^2 - y^2$.

- (3) Sketch a contour map of the function $f = e^{y/x}$.

- (4) Compute the following limits:

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + \sin^2(y)}{2x^2 + y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{y^4 \sin(xy)}{x^2 + y^2}$

For Exercises 5 - 8, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(5) $f(x, y) = e^{xy} + xy$

(6) $f(x, y) = x^4$

(7) $f(x, y) = \frac{x+1}{y+1}$

(8) $f(x, y) = \ln(xy) + x^y$

For Exercises 9 and 10, calculate all four second-order partial derivatives of the function and verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

(9) $f(x, y) = x^2 + y^2$

(10) $f(x, y) = \cos(x + y)$

- (11) The ideal gas law is $PV = cT$ where P is the pressure, V is the volume, T is the temperature and c is a constant. Show that

$$\frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = -1.$$

(Treat each variable as a function of the other two.)

- (12) Find the linearization $L(x, y)$ of the function $f(x, y) = x\sqrt{y}$ at the point $(1, 4)$.