HOMEWORK 8, DUE TUESDAY NOVEMBER 7TH

- (1) Write down explicit triple integrals for the mass and the z-coordinate of the center of mass of the hemisphere $x^2 + y^2 + z^2 \le 1$, $z \ge 0$, with density $\rho = \sqrt{x^2 + y^2 + z^2}$. You do not have to evaluate these integrals (!).
- (2) Compute the integral $\int \int \int_R e^{x^2+y^2+z^2} dV$ where R is the region inside the sphere $x^2+y^2+z^2=9$ and within the first octant (i.e. $x\geq 0,\,y\geq 0,\,z\geq 0$). A numerical answer is acceptable.
- (3) Find the volume of a wedge cut from a ball of radius R by two planes which intersect on a diameter of the ball at an angle of $\pi/6$. (This volume is like a section of an orange.)
- (4) Compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transformation $x=(u^2-v^2)/2, y=(u^2+v^2)/2.$
- (5) Use the transformation $x = \sqrt{2}u \sqrt{2/3}v$, $y = \sqrt{2}u + \sqrt{2/3}v$ to compute the integral $\int \int_R (x^2 xy + y^2)^{1/2} dA$ where R is the region bounded by the ellipse $x^2 xy + y^2 = 2$.
- (6) Match the vector field plots on the second page to the following vector fields (as usual the x-axis is the horizontal axis in these plots):
 - (I) $\vec{F} = (x 1, x + 3)$
 - (II) $\vec{F} = (y, x)$
 - (III) $\vec{F} = (\sin(x), 1)$
 - (IV) $\vec{F} = (y, 1/x)$
- (7) Compute the scalar line integral $\int_C y \, ds$ where C is the curve $x = y^2$ for $y \in [0,3]$.
- (8) Compute the scalar line integral $\int_C x/y \, ds$ where C is the curve $x = t^3$, $y = t^4$, $t \in [1/2, 1]$.
- (9) Compute the scalar line integral $\int_C xy^3 ds$ where C is the curve $x = 3\sin(t)$, $y = 3\cos(t)$, z = 4t for $t \in [0, \pi/2]$.
- (10) Is the vector line integral $\int_C \vec{F} \cdot d\vec{s}$ positive, negative, or zero for the \vec{F} and C shown below? (The curve C is in blue.)



