

Math 3298 Worksheet 43:

Group members (2 to 4): _____

- (1) Suppose the magnitude of a 3D vector field $F = (f_1, f_2, f_3)$ is bounded at each point in space by $|F| = \sqrt{f_1^2 + f_2^2 + f_3^2} \leq |z|$. Use the divergence theorem to find an upper bound on the magnitude of $\int \int \int_R \operatorname{div} F \, dV$ where R is the unit cube $x \geq 0, x \leq 1, y \geq 0, y \leq 1, z \geq 0, z \leq 1$.

- (2) For the vector field $G = (-2xz, 0, y^2)$:
- (a) Compute the curl of G .

- (b) Show that $\oint_C G \cdot dr = 0$ for any simple closed smooth curve on the sphere $x^2 + y^2 + z^2 = 4$.