Math 3298 Practice final exam problems

The actual exam will consist of six to eight required questions and possibly an optional extra credit question.

(1) Find the integral of the function $f(x, y) = 2x\sqrt{y^2 - x^2}$ over the triangle $T = \{(x, y) \mid 0 \leq y \leq 2, 0 \leq x \leq y\}$

(2) Find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 1$.

(3) Compute the integral $\displaystyle\int\int\int_R \sqrt{x^2 + y^2}\ dV$ where R is the region inside the cylinder $x^2 + y^2 = 25$ and between $z = -1$ and $z = 4$.

(4) Find the volume of the solid bounded by the planes $z = x$, $y = x$, $x + y = 2$, and $z = 0$.

(5) Change the order of integration of $\int_0^2 \int_{Arctan(x)}^{Arctan(\pi x)} dydx$ and evaluate the integral.

(6) Compute the integral $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2}\ dz\ dy\ dx$ by changing to cylindrical coordinates.

(7) This difficult a problem would be extra credit: Assuming that $\beta \in (0, \pi/2)$ and $a > 0$, compute the following integral

$$\int_0^{a \sin \beta} \int_{y \cot \beta}^{\sqrt{a^2-y^2}} \int_0^1 \ln\left(x^2 + y^2\right)\ dz\ dx\ dy$$

(8) Reverse the order of integration for the integral $\int_0^1 \int_x^1 \int_0^{y^2} f(x, y, z) dzdydx$.

(9) Compute the value of the vector line integral $\int_C \vec{F} \cdot d\vec{r}$ where C is the path $(4 - 3^t, -2 + 2t, \pi t)$, $t \in [0, 1]$, and $\vec{F} = (2x \cos z - x^2, z - 2y, y - x^2 \sin z)$.

(10) Find the linearization of $f(x, y)$ at $(x, y) = (0, 1)$ if $f = h(u(x, y), v(x, y))$ and $\text{grad}(h)|_{(1,1)} = (\frac{\partial h}{\partial u}, \frac{\partial h}{\partial v})|_{(1,1)} = (2, 3)$, $u(x, y) = x + y$, and $v(x, y) = y^2$.

(11) Find the surface area of the torus parameterized by $x = (2 + \cos(v)) \cos(u)$, $y = (2 + \cos(v)) \sin(u)$, $z = \sin(v)$, with $u \in [0, 2\pi]$ and $v \in [0, 2\pi]$.

(12) Find the maxima and minima of $f(x, y) = \frac{1}{x} + \frac{2}{y}$ on the set $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

(13) Find the volume of the solid wedge bounded by the planes $z = 0$ and $z = -2y$ and the cylinder $x^2 + y^2 = 4$ (with $y \geq 0$).

(14) Use Green's Theorem to find the smooth, simple, closed and positively oriented curve in the plane for which the line integral $\oint (\frac{x^2y}{4} + \frac{y^3}{3})dx + xdy$ has the largest possible value.

(15) Compute the value of $\int\int_S (\bigtriangledown \times \vec{F}) \cdot \vec{n}\ dS$ where S is the upper half of the ellipsoid $4x^2 + 9y^2 + 36z^2 = 36$, $z \geq 0$, with upward pointing normal, and $\vec{F} = (y, x^2, (x^2 + y^2)^{3/2} e^{xyz})$.

(16) Let $\vec{r}(t)$ be a curve in space with unit tangent, normal, and binormal vectors $\vec{T}$, $\vec{N}$, and $\vec{B}$. Show that $\frac{d\vec{B}}{dt}$ is perpendicular to $\vec{T}$.

(17) Compute the flux integral $\int\int_S \vec{F} \cdot \vec{n}\ dS$ where S is the graph of $z = 1 - x^2 - y^2$, with upward normal, for $z \geq 0$, and with $\vec{F} = (xz, yz, 2z^2)$.

(18) Use the divergence theorem to compute the flux of $\vec{F} = (z^5 + x, \cos(xz), z^2)$ through the surface bounded by $z = 0$ and $z = 1 - x^2 - y^2$.