Math 3298 Practice Midterm

This practice test is longer than the actual exam.

(1) (a) Use the formula $\kappa = \dfrac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$ to show that for a parameterized plane curve $(x(t), y(t))$ the curvature is

$$\kappa = \frac{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}{|\dot{x}^2 + \dot{y}^2|^{3/2}}$$

(b) Use the result of part (a) to compute the curvature of $x(t) = 1 + t^3$, $y(t) = t + t^2$.

(2) Classify the critical points of $f(x, y) = 2y^2 + 2xy - y - x^3 + x + 1$.

(3) Compute the limit $\dfrac{x^2 + y\sin(y)}{x^2 + y^2}$ if it exists, or show why it does not exist.

(4) Find the curvature of $\vec{r}(t) = (t^2, t^3, 2t^3)$ at $t = 1$.

(5) Use the linearization of the function $f(x, y) = x + \ln(xy)$ at $(x, y) = (2, 1/2)$ to find an approximate value for $f(1.9, .4)$.

(6) Find three positive numbers x, y, and z such that $x + 2y + 3z = 7$ and for which the function $f(x, y, z) = x^2y^2z^3$ is maximized.

(7) Use the chain rule to compute $\frac{\partial z}{\partial t}$ at $t = 2$ if $z = \sin(xy)\sin(y)$ and $x = 1/t$, $y = f(t)$ where $f'(2) = 3$ and $f(2) = \pi$.

(8) Find the directions in which the directional derivative of $f(x, y) = x^2 + 2y^2 - 4y$ at the point $(1, 1)$ has the value 1.