Math 3298 Practice Midterm

This practice test is longer than the actual exam.

1. (a) Use the formula \( \kappa = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} \) to show that for a parameterized plane curve \( (x(t), y(t)) \) the curvature is

\[
\kappa = \frac{|\dot{x}y - \dot{y}x|}{|\dot{x}^2 + \dot{y}^2|^{3/2}}
\]

(b) Use the result of part (a) to compute the curvature of \( x(t) = 1 + t^3, y(t) = t + t^2 \).

2. Classify the critical points of \( f(x, y) = 2y^2 + 2xy - y - x^3 + x + 1 \).

3. Compute the limit \( \lim_{x \to 0} \frac{x^2 + y \sin(y)}{x^2 + y^2} \) if it exists, or show why it does not exist.

4. Find the curvature of \( \vec{r}(t) = (t^2, t^3, 2t^3) \) at \( t = 1 \).

5. Use the linearization of the function \( f(x, y) = x + \ln(xy) \) at \( (x, y) = (2, 1/2) \) to find an approximate value for \( f(1.9, .4) \).

6. Find three positive numbers \( x, y, \) and \( z \) such that \( x + 2y + 3z = 7 \) and for which the function \( f(x, y, z) = x^2 y^2 z^3 \) is maximized.

7. Use the chain rule to compute \( \frac{\partial z}{\partial t} \) at \( t = 2 \) if \( z = \sin(xy) \sin(y) \) and \( x = 1/t, y = f(t) \) where \( f'(2) = 3 \) and \( f(2) = \pi \).

8. Find the directions in which the directional derivative of \( f(x, y) = x^2 + 2y^2 - 4y \) at the point \( (1, 1) \) has the value 1.