

Math 3298 Worksheet 13: More double integrals

Group members (1 to 3): _____

(1) Convert the following integral to polar coordinates and evaluate it:

$$\int_{-1}^{-1/2} \int_{-\sqrt{3}x}^{\sqrt{4-x^2}} x^2 y^2 dy dx + \int_{-1/2}^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} x^2 y^2 dy dx + \int_1^2 \int_0^{\sqrt{4-x^2}} x^2 y^2 dy dx$$

- (2) Find the center of mass of the lamina defined by $y \geq x^2$, $y \leq 4$, $x \geq 0$ with $\rho(x, y) = x$. The center of mass is $(M_y/m, M_x/m)$ where m is the total mass ($m = \int \int_R \rho \, dA$) and $M_y = \int \int_R x\rho \, dA$, $M_x = \int \int_R y\rho \, dA$. Sketch the integration region and indicate the center of mass.