A Mathematical Coloring Book

by Marshall Hampton

Dedicated to Violet Hampton

Version 0.94

Send comments and suggestions to hamptonio@gmail.com

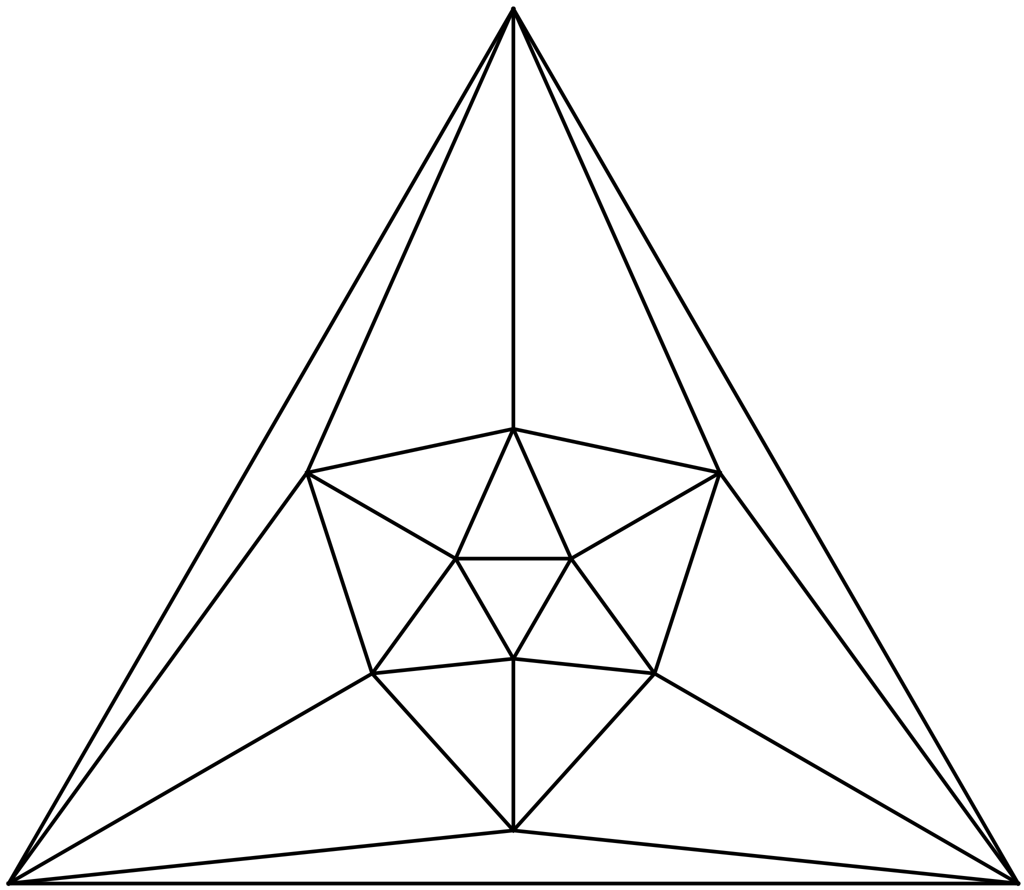

Figure 1: Schlegel projection of an icosahedron

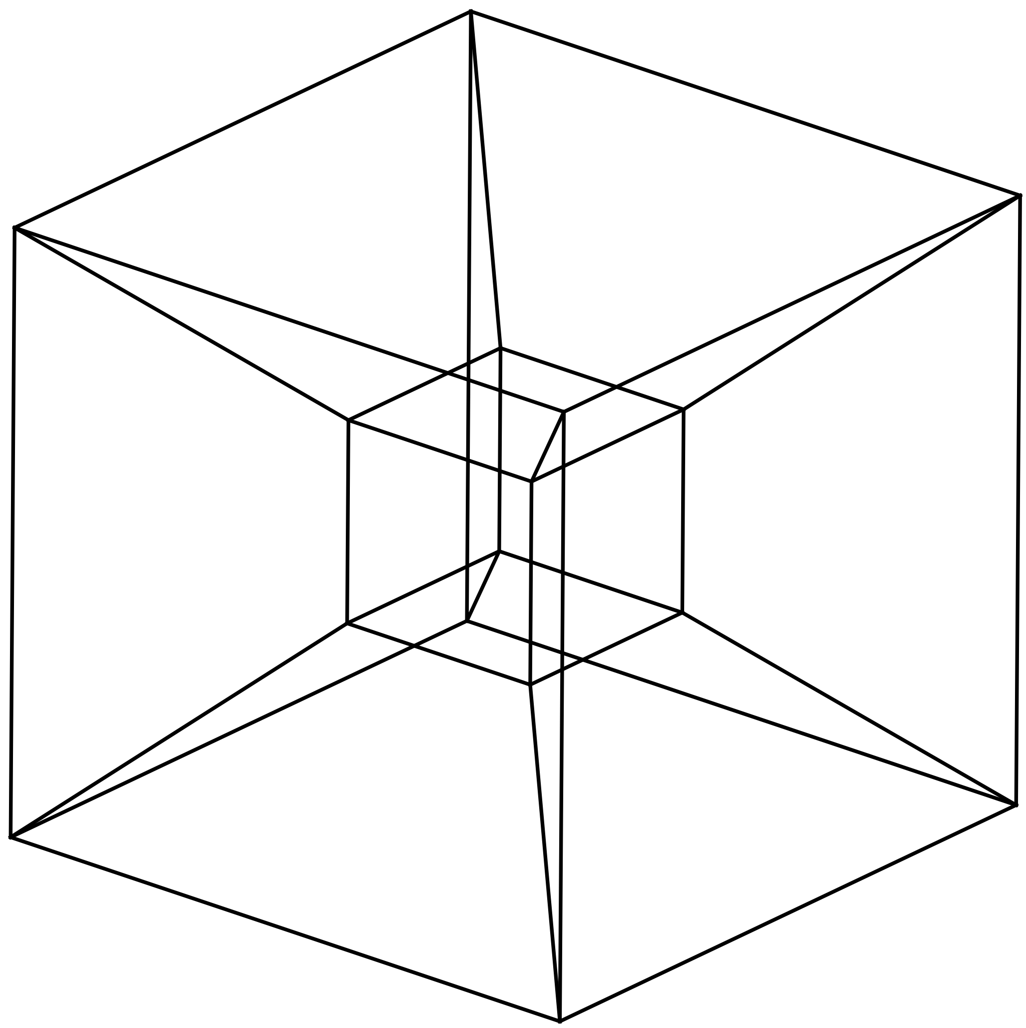

Figure 2: Schlegel projection of an hypercube (tesseract)

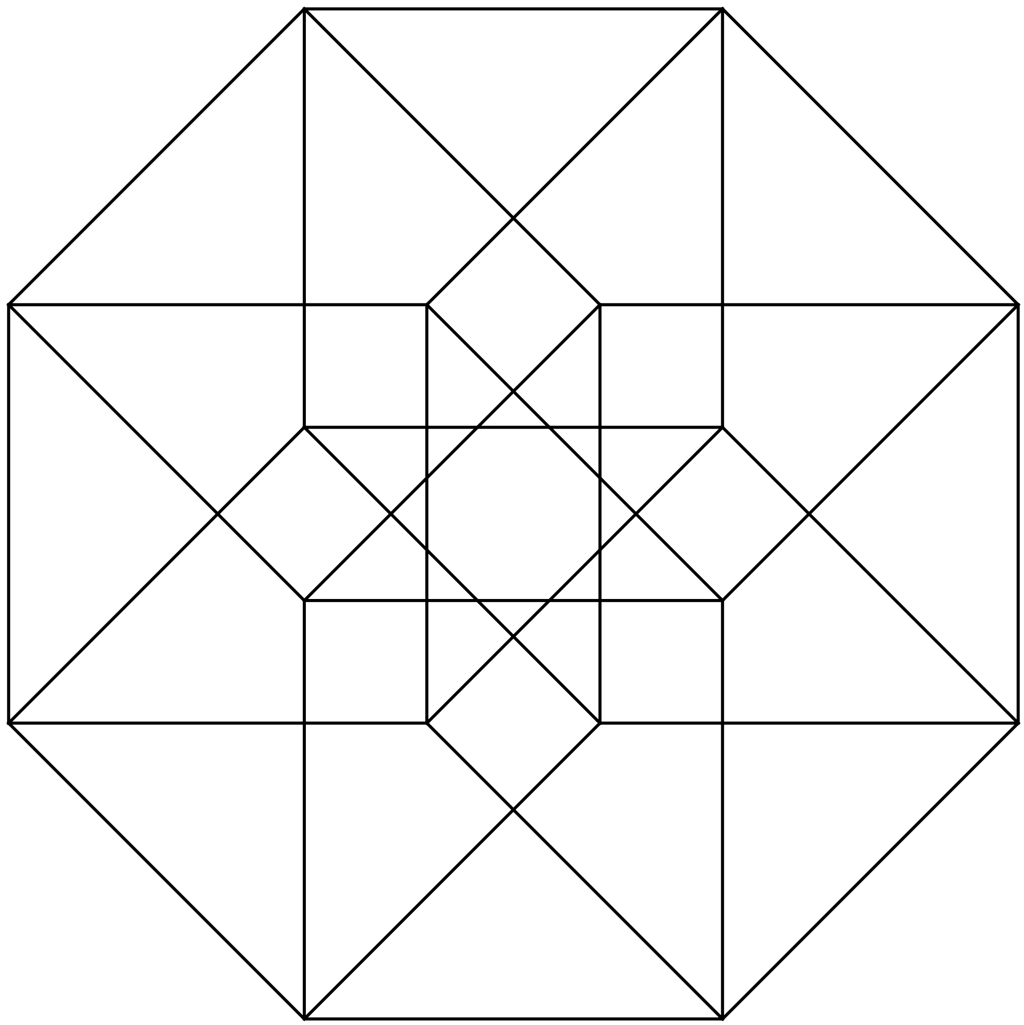

Figure 3: Orthogonal hypercube projection

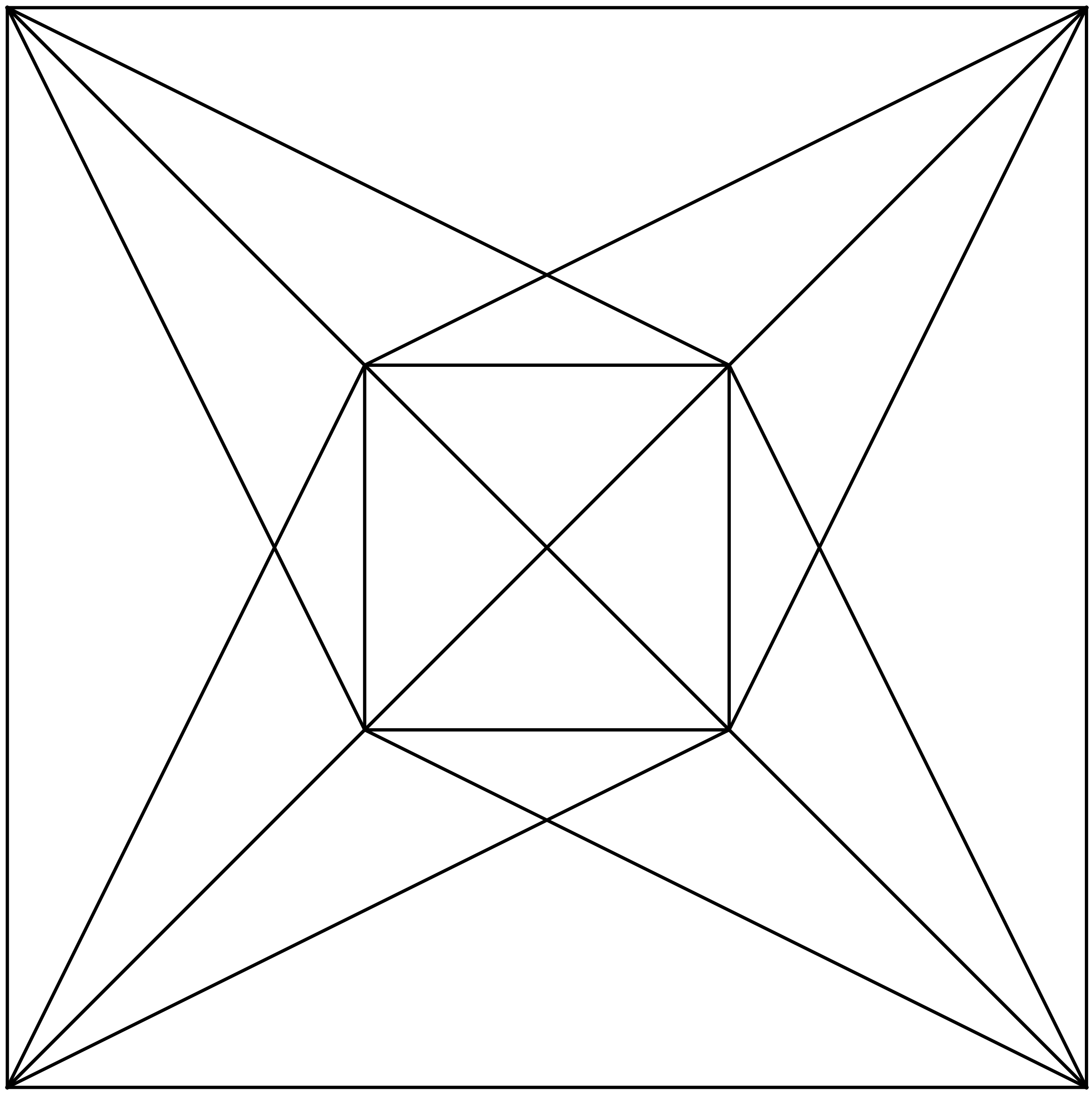

Figure 4: Four-dimensional cross-polytope Schlegel projection

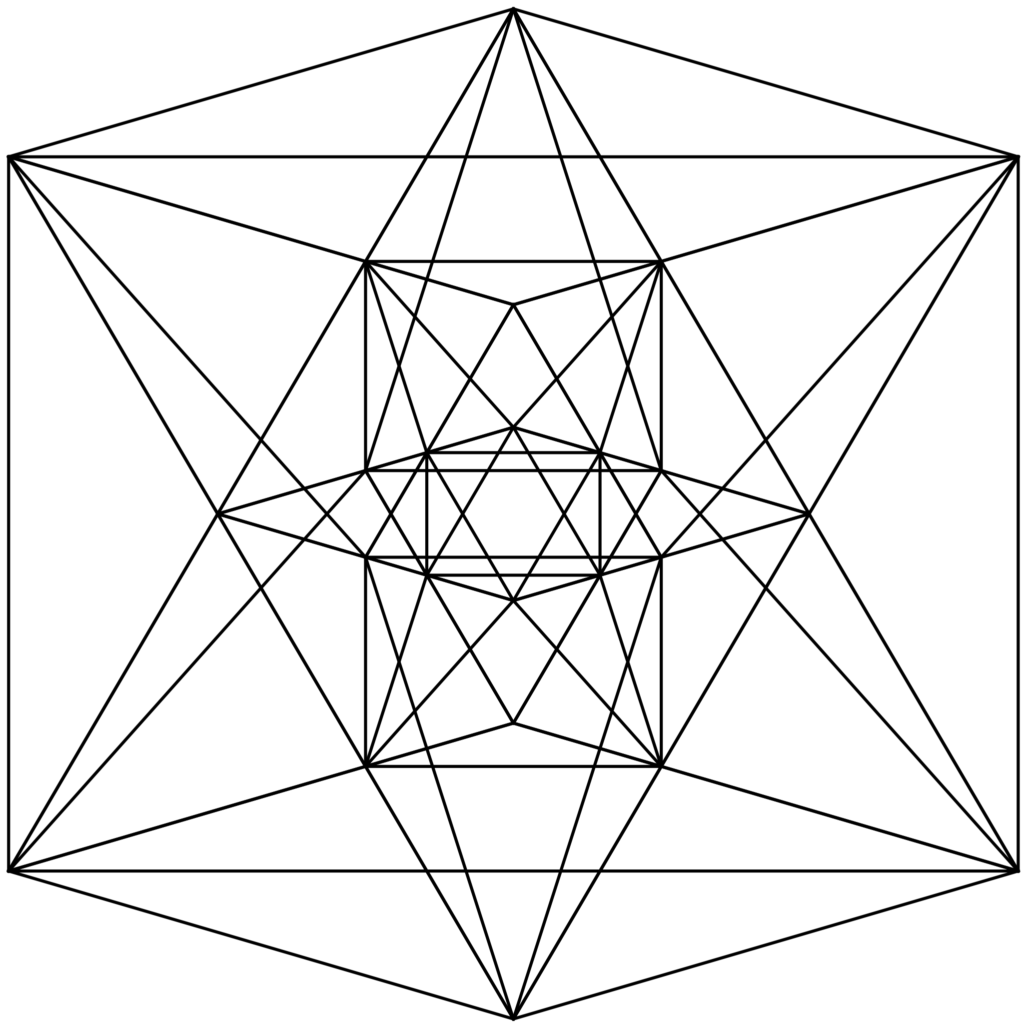

Figure 5: Schlegel projection of the twenty-four cell

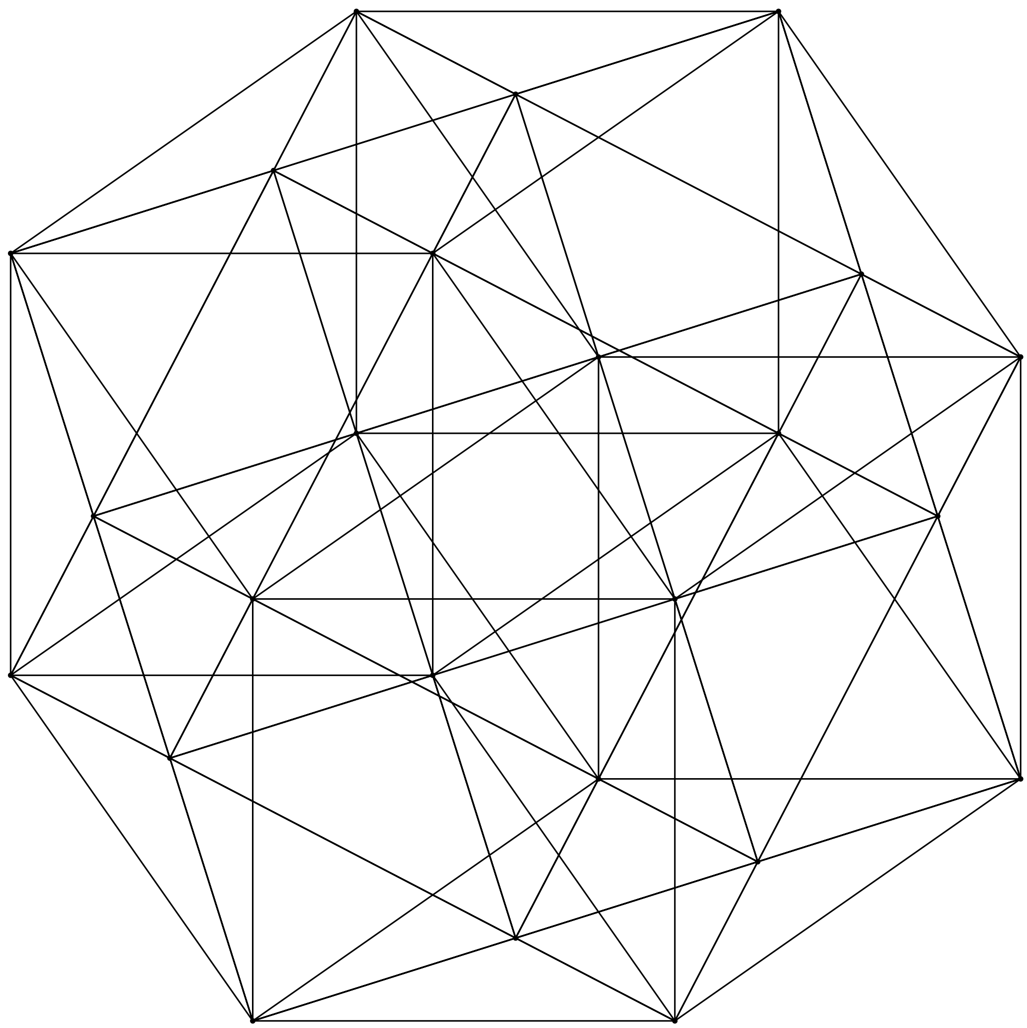

Figure 6: Orthogonal projection of the twenty-four cell

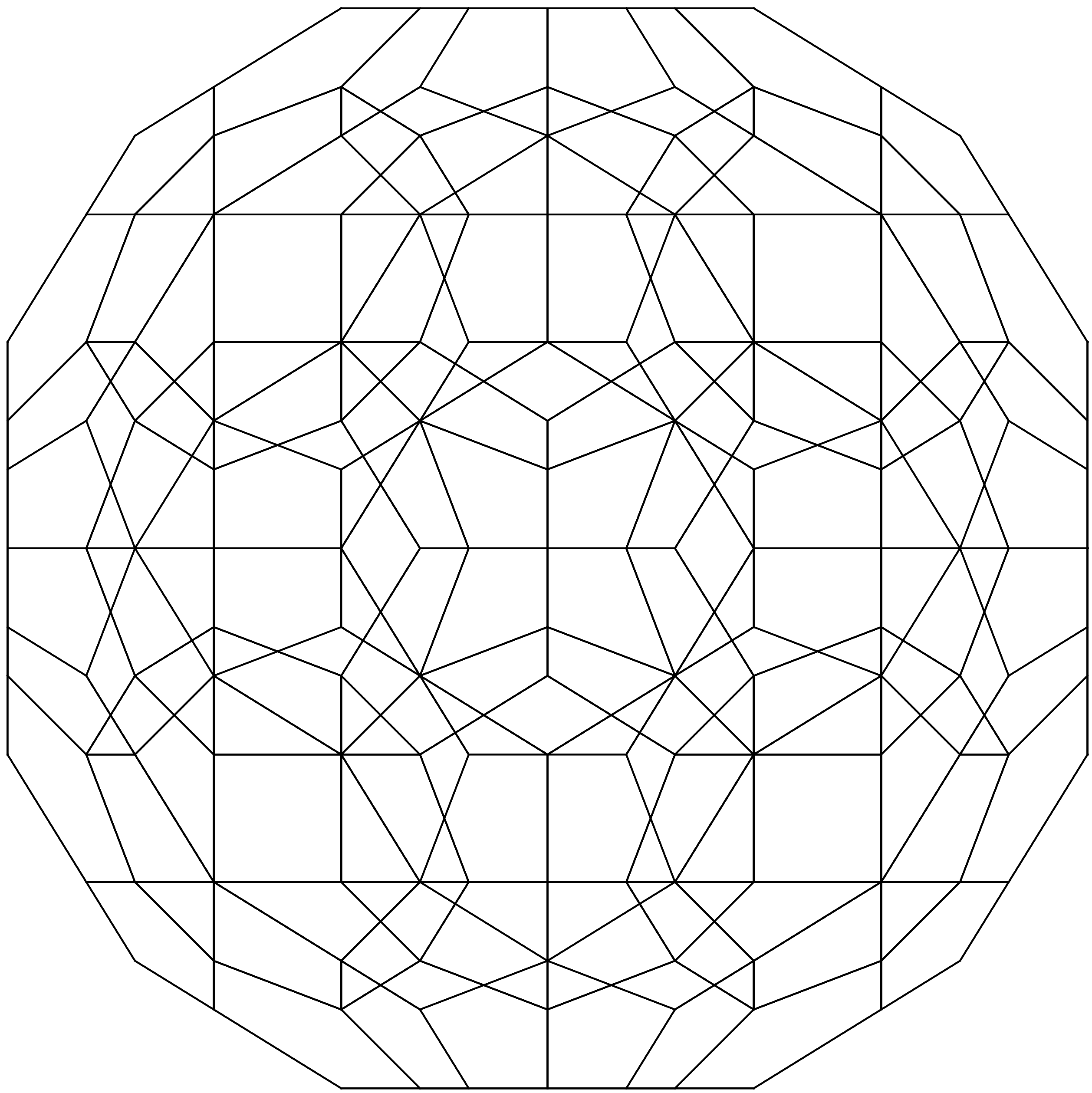

Figure 7: Orthogonal projection of the one hundred and twenty cell

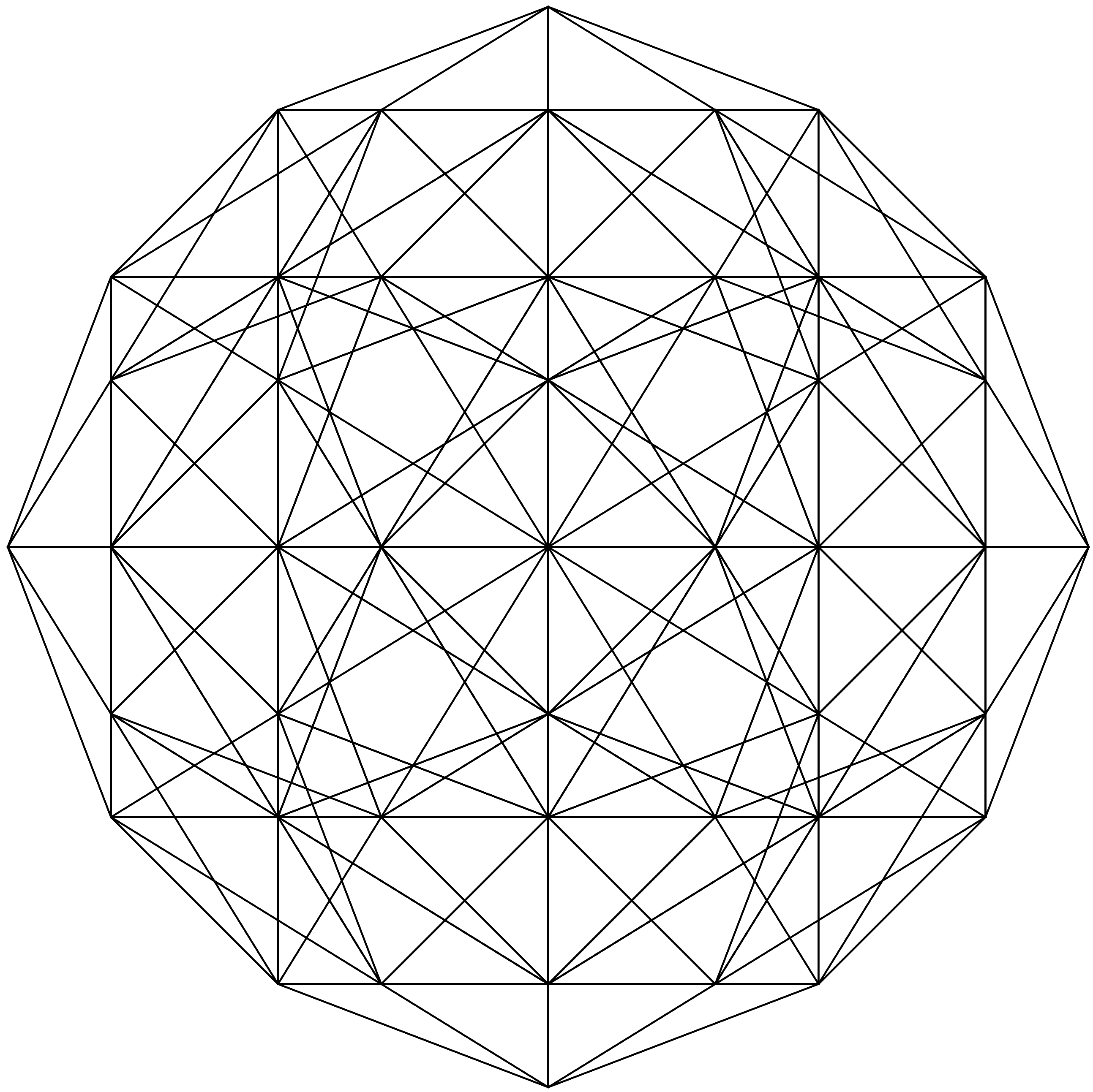

Figure 8: Orthogonal projection of the six hundred cell

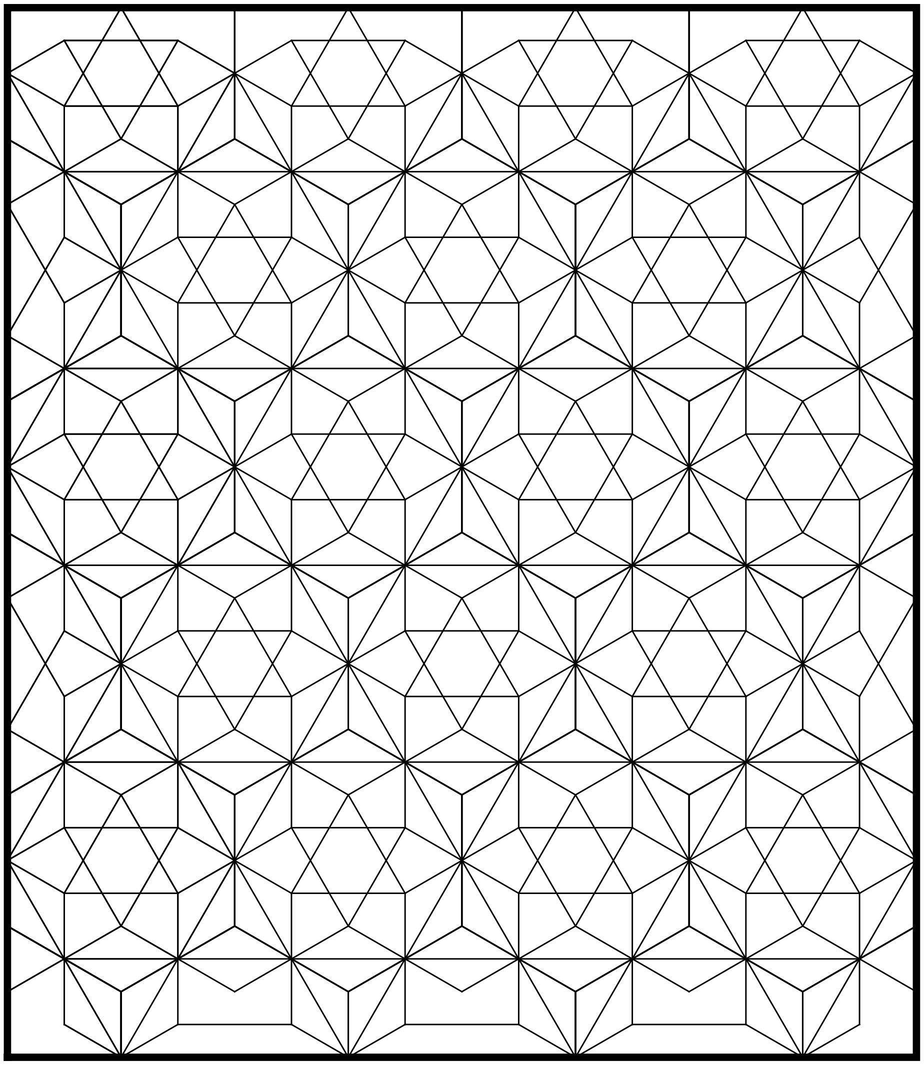

Figure 9: Hexagonal tiling from projection

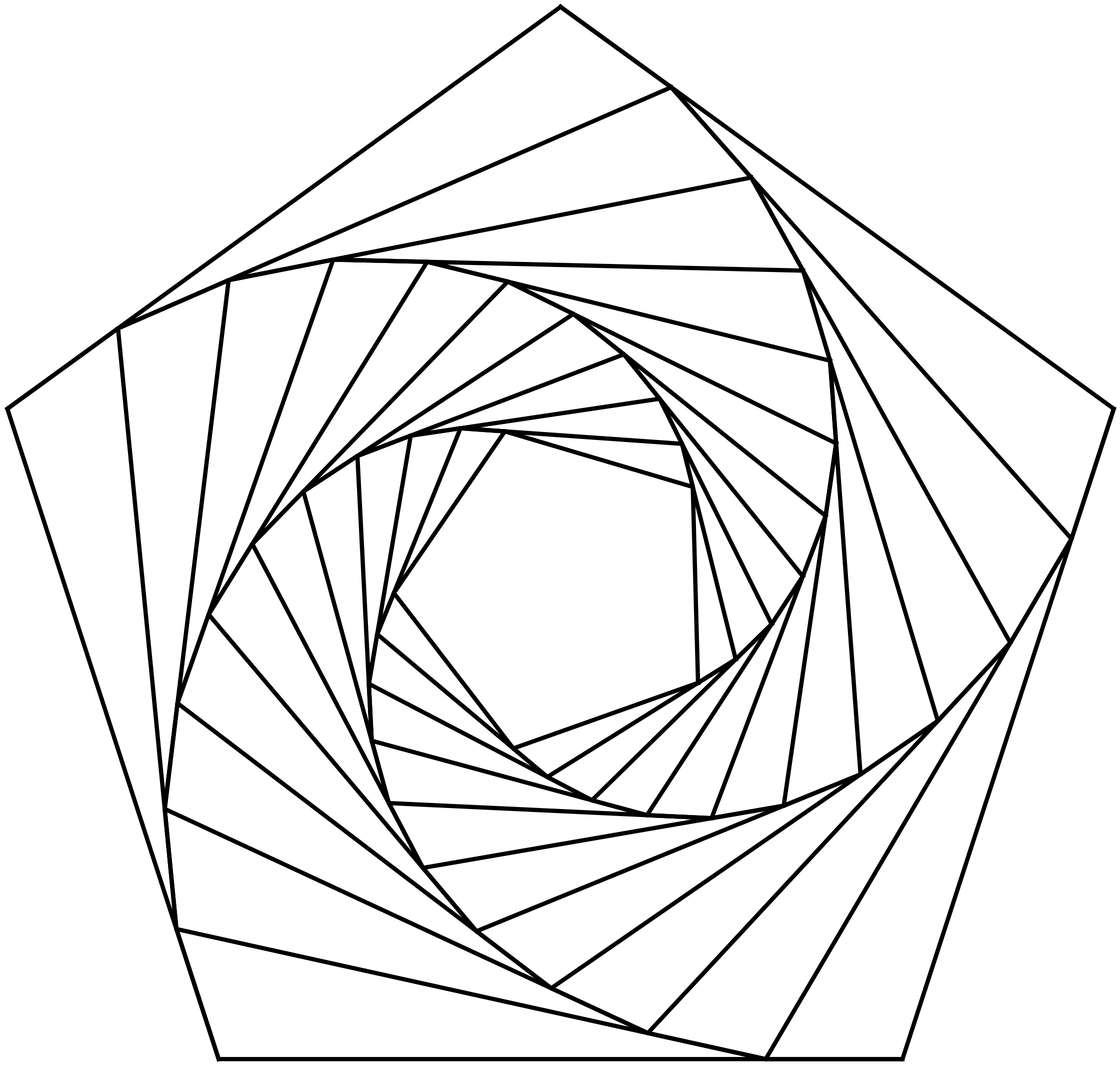

Figure 10: 5-body pursuit paths

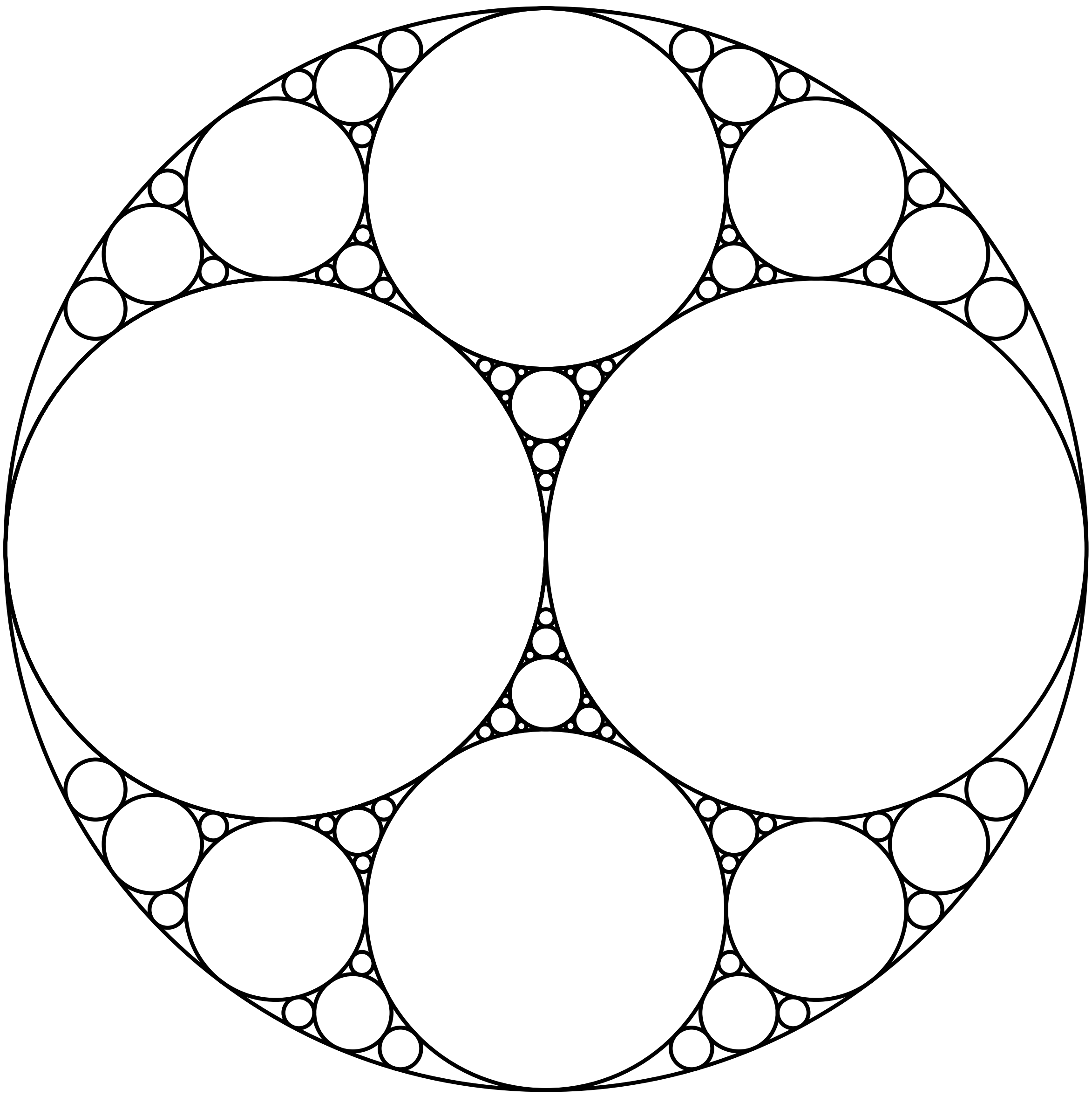

Figure 11: Integral curvature Apollonian circle packing

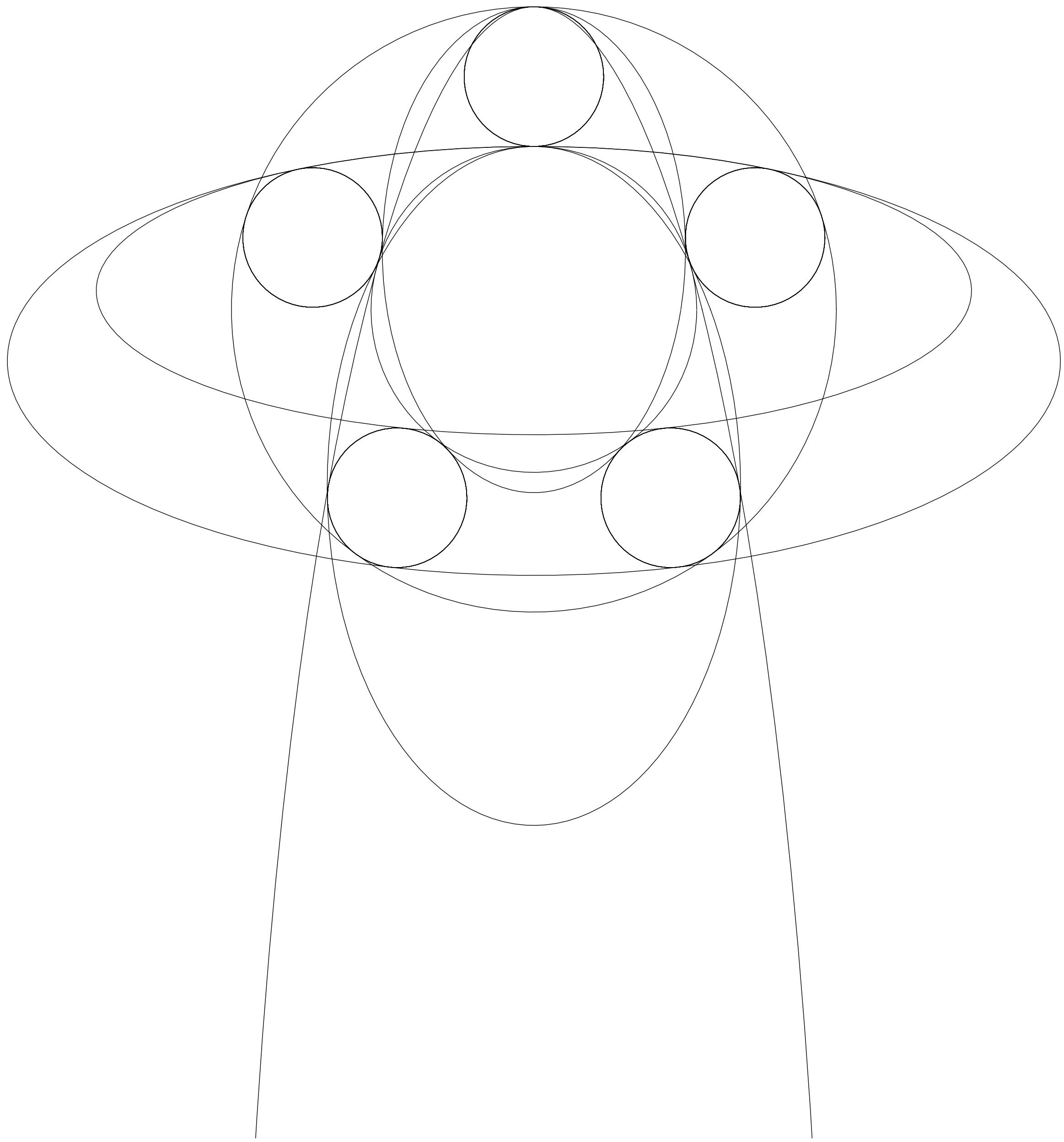

Figure 12: Tangent conics to five circles

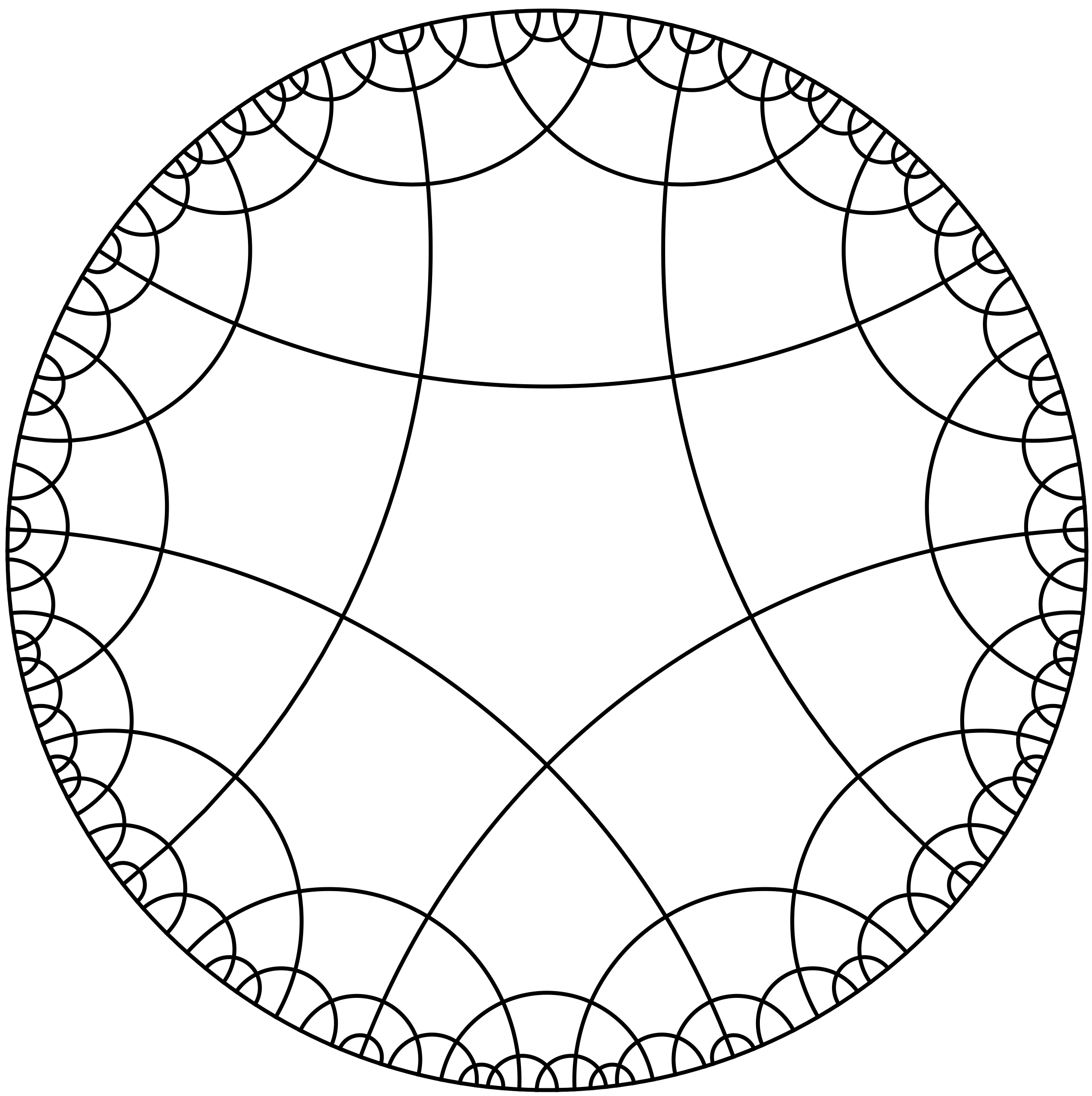

Figure 13: Hyperbolic geodesics

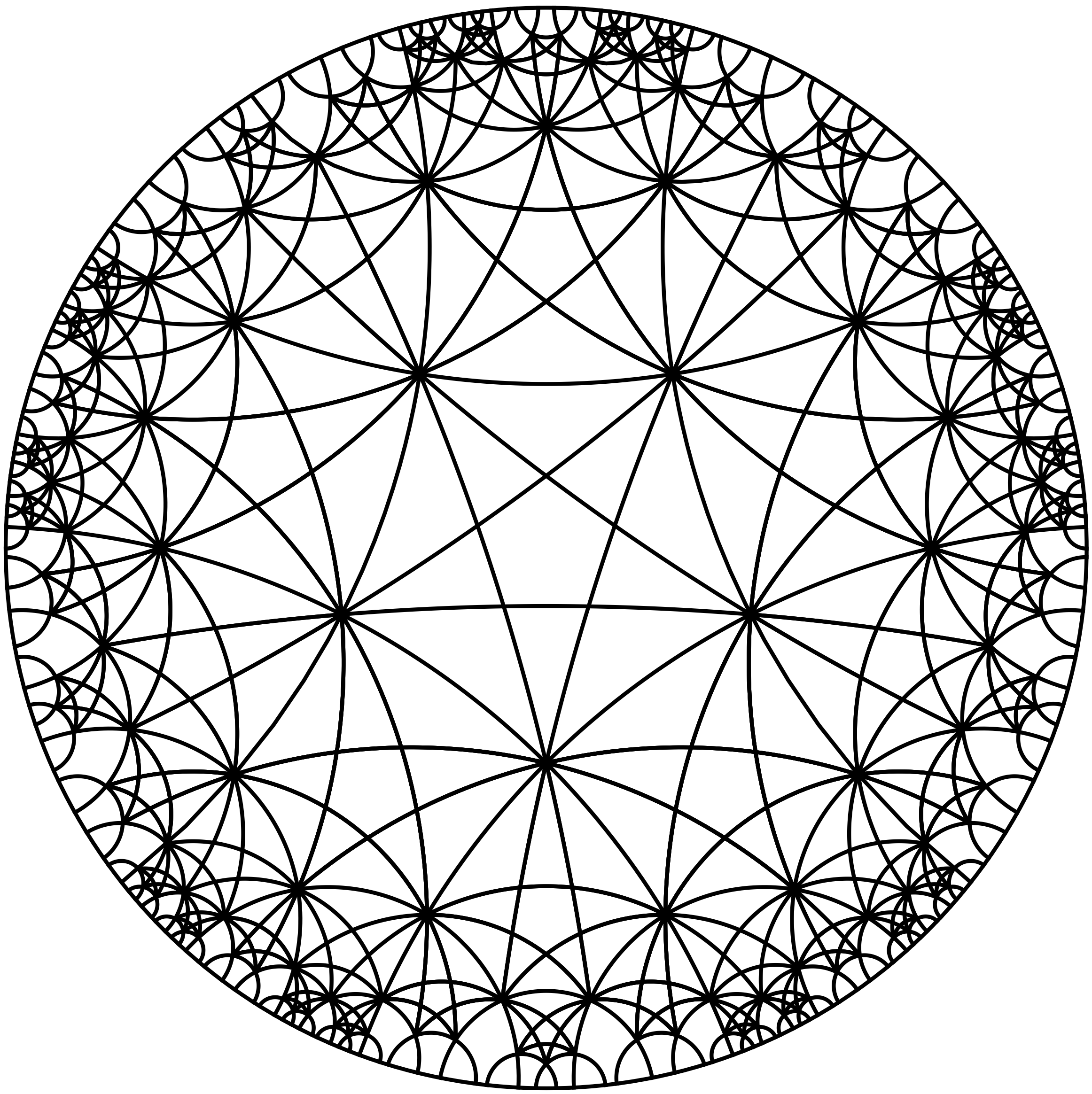

Figure 14: Hyperbolic geodesic star tiling

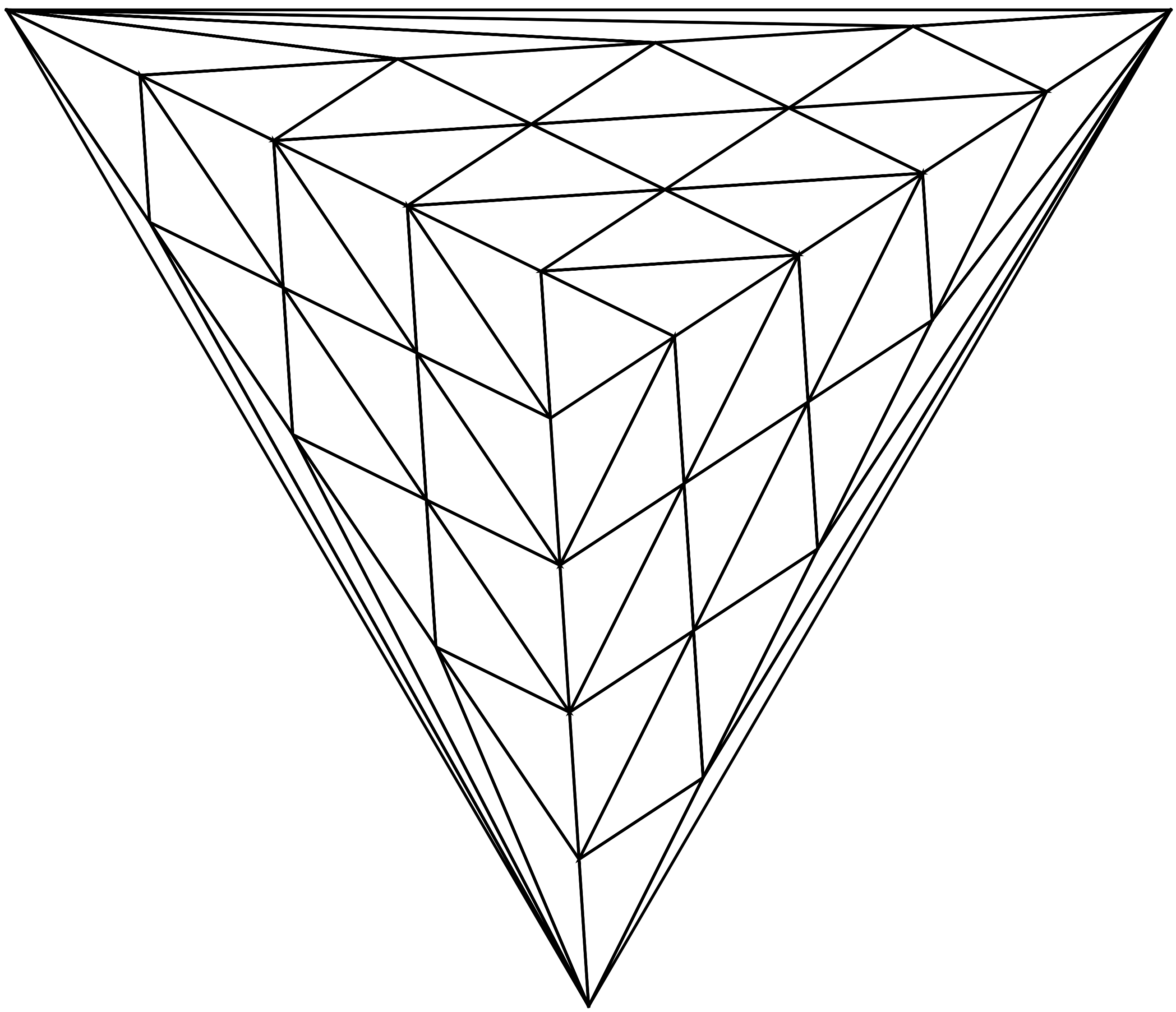

Figure 15: Gröbner fan # 1.

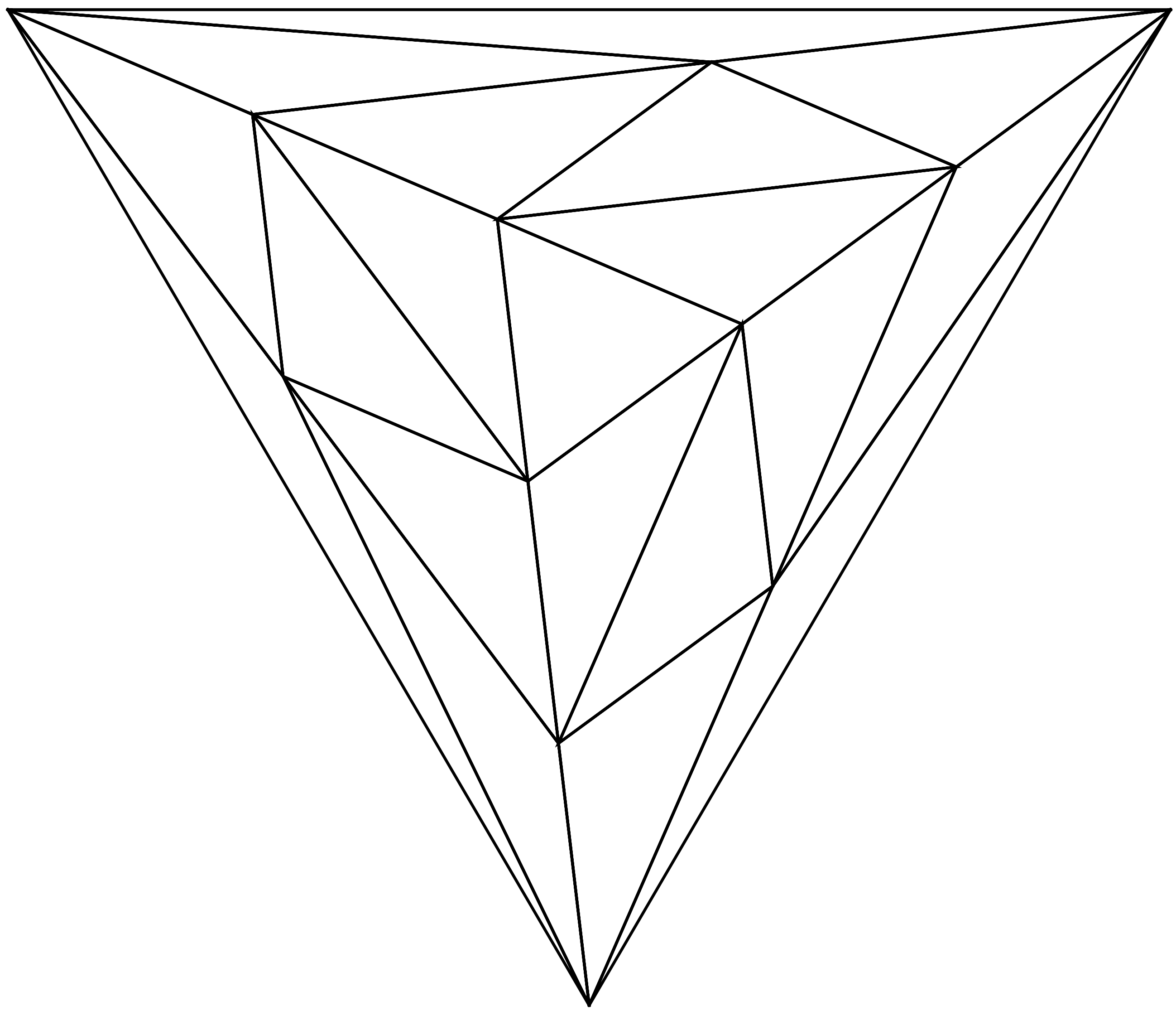

Figure 16: Gröbner fan # 2.

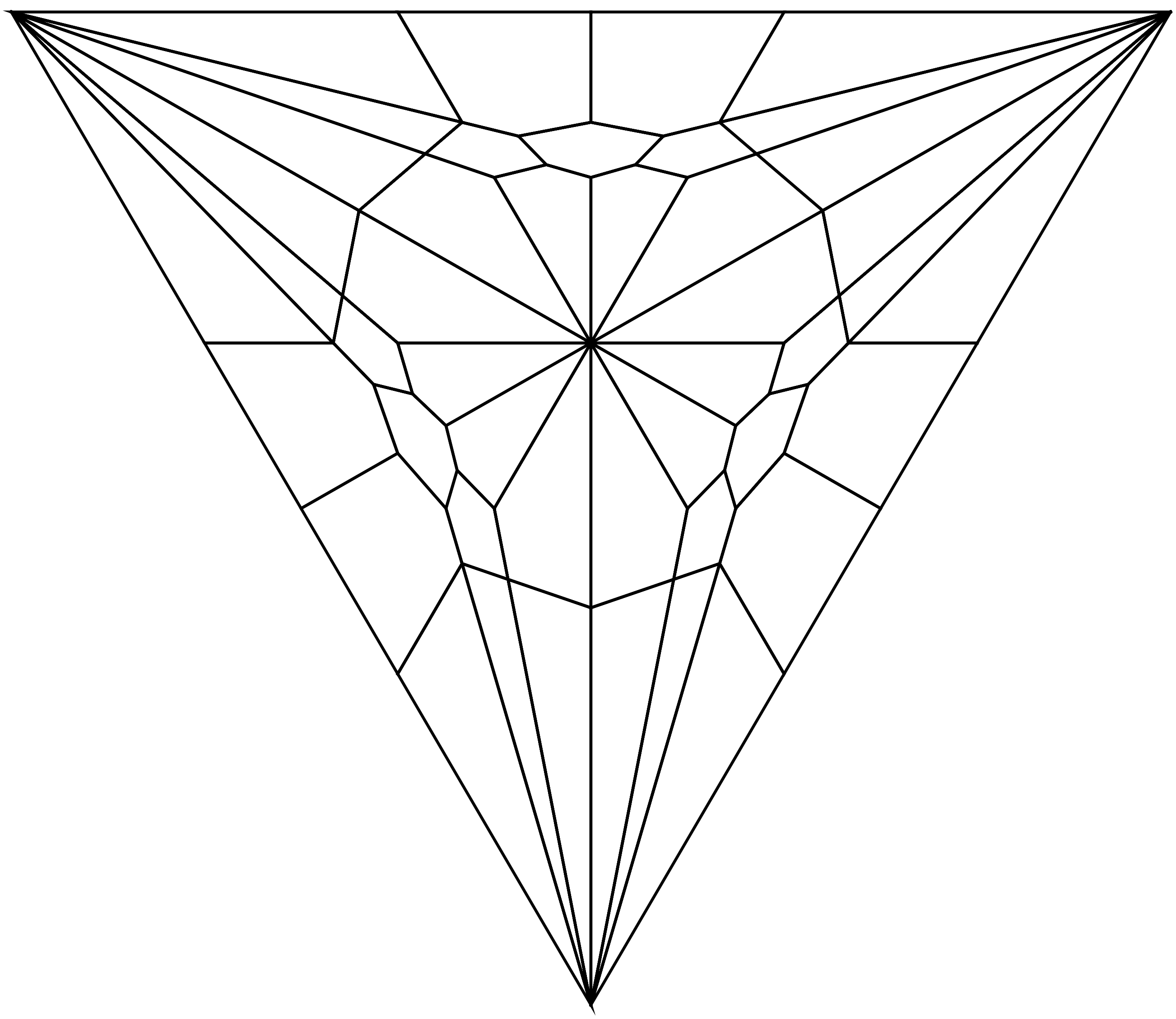

Figure 17: Gröbner fan of the ‘3-vortex problem’.

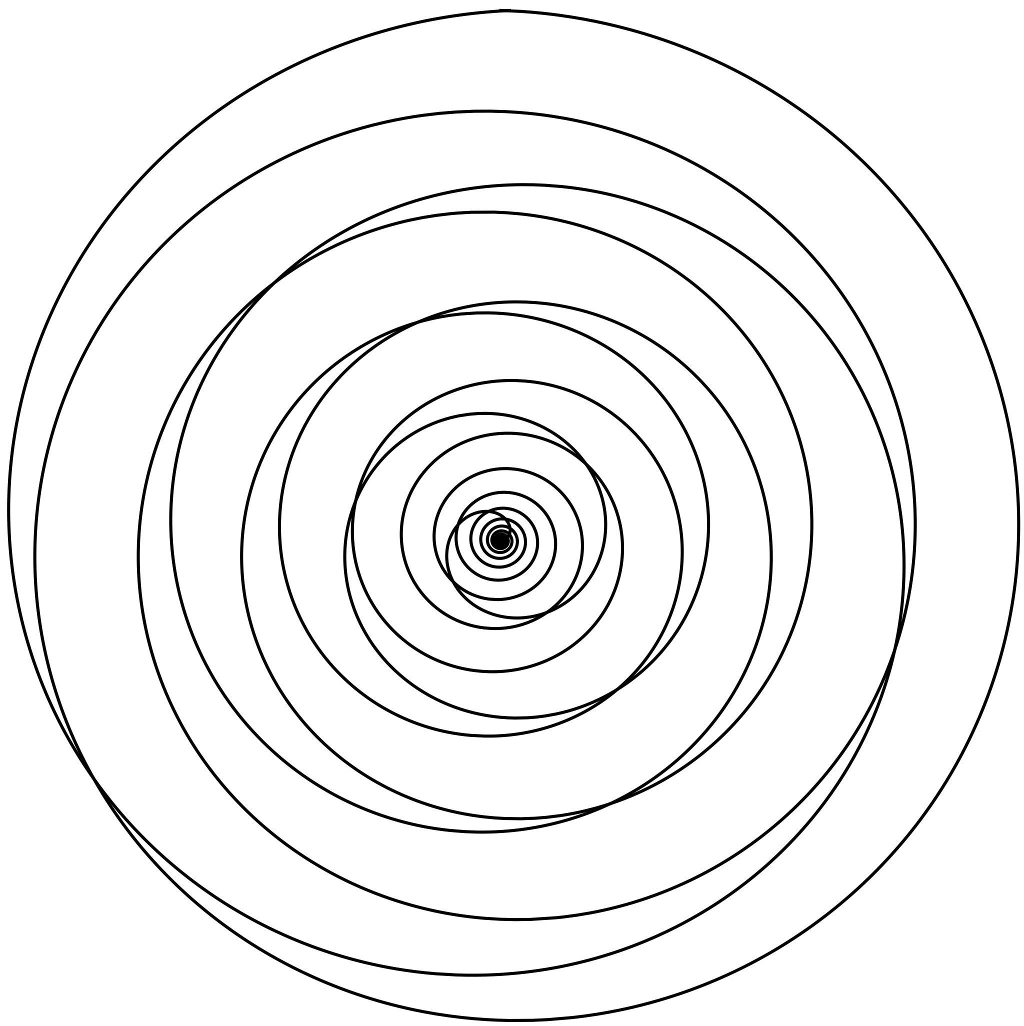

Figure 18: Archimedean and Exponential Spirals

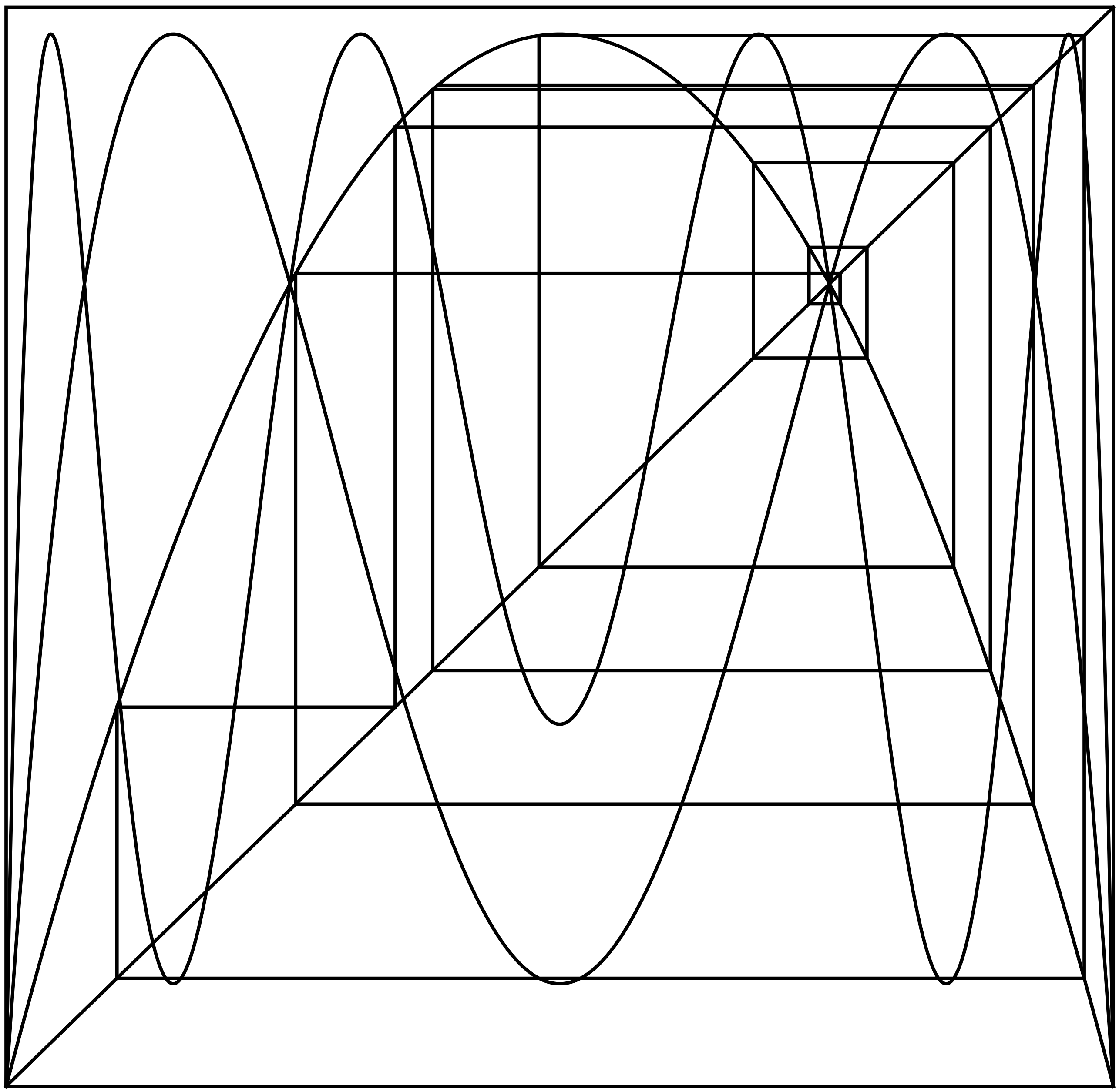

Figure 19: Cobweb diagram of the logistic map # 1.

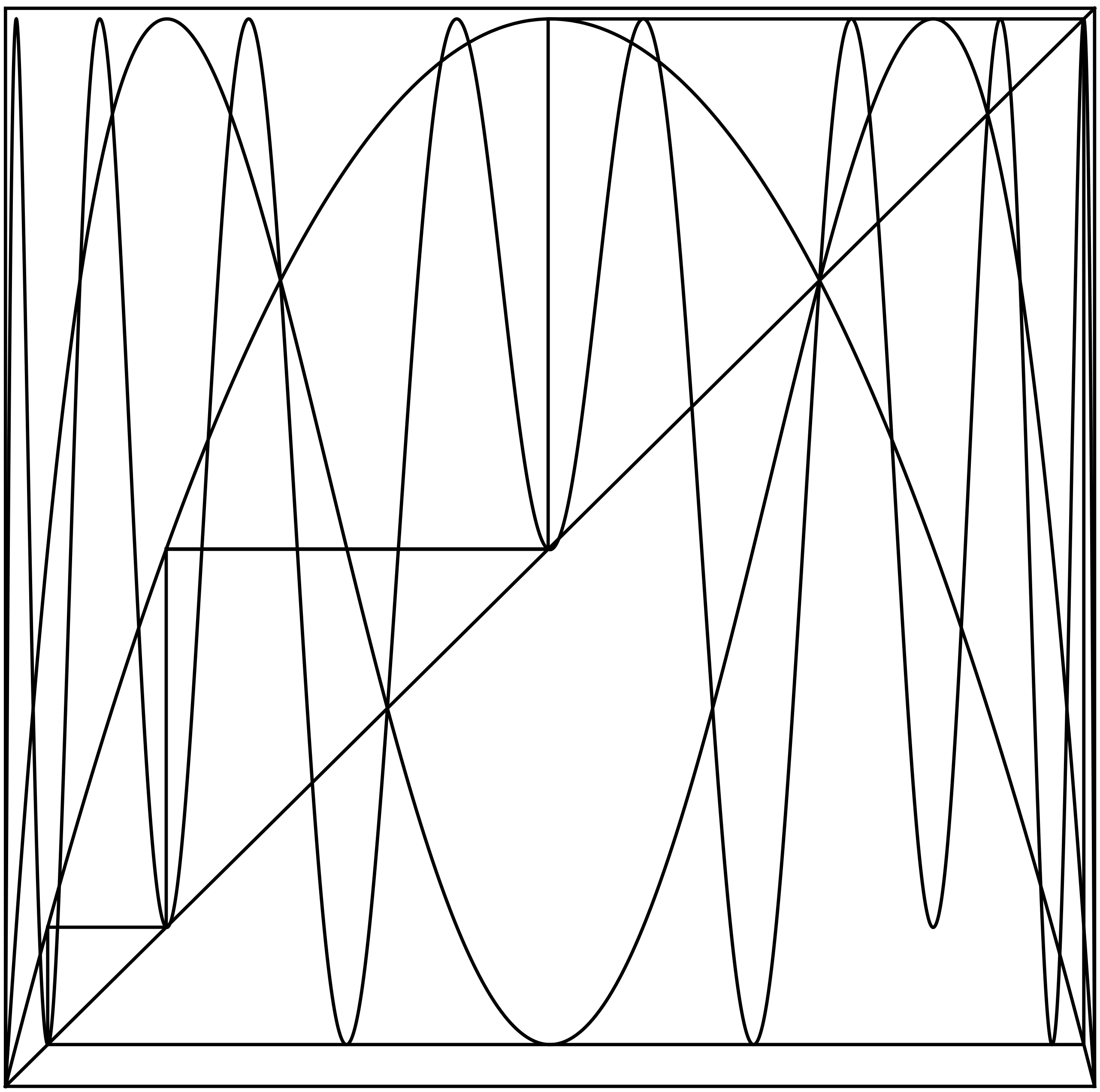

Figure 20: Cobweb diagram of the logistic map # 2.

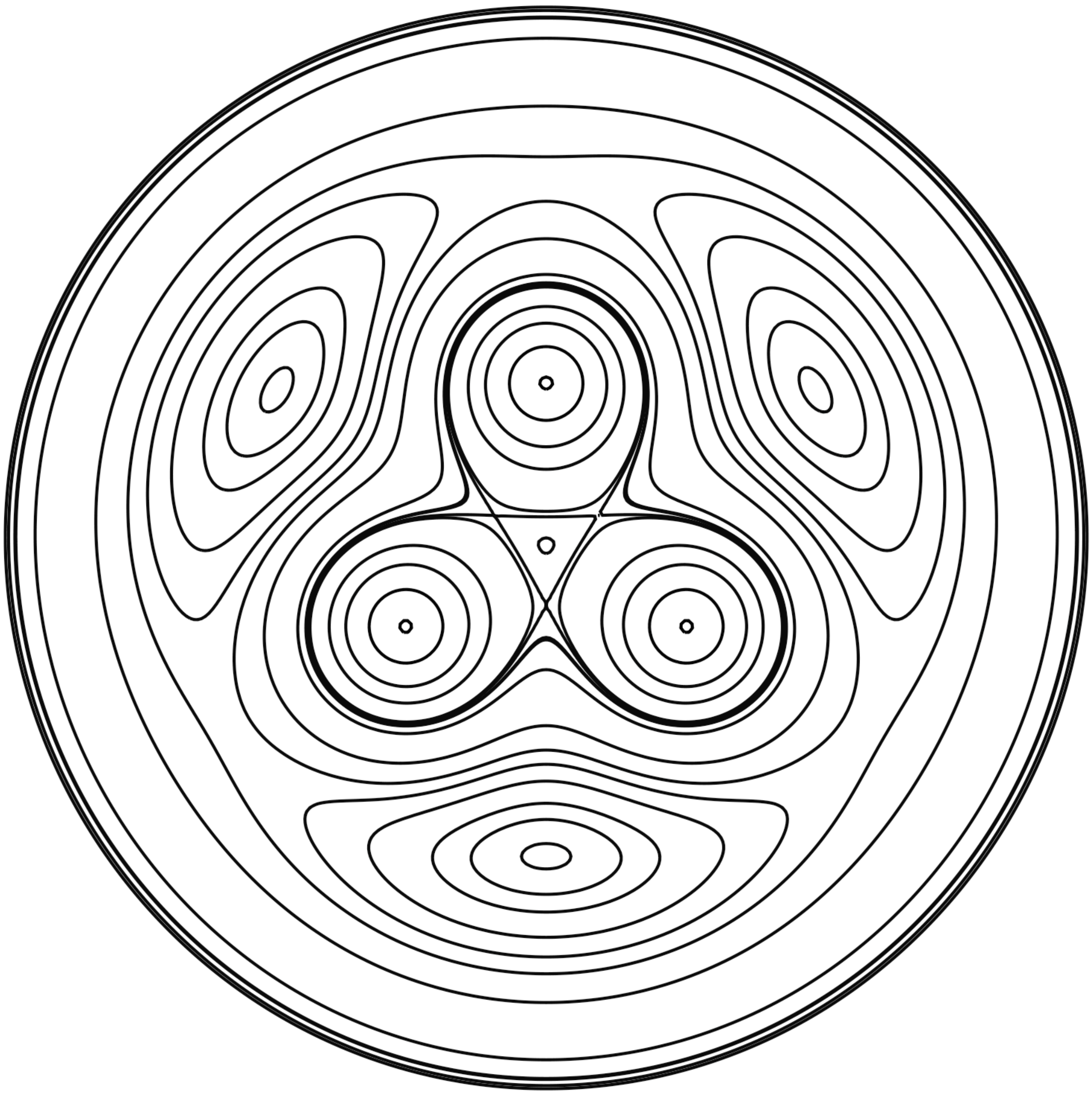

Figure 21: Equipotential lines of the three-body problem (equal masses).

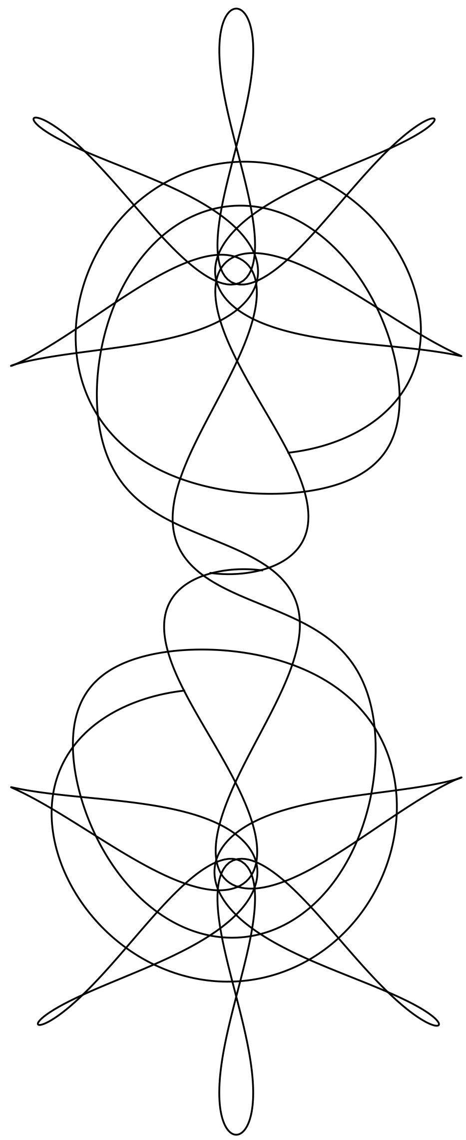

Figure 22: Orbits in the circular three-body restricted problem.

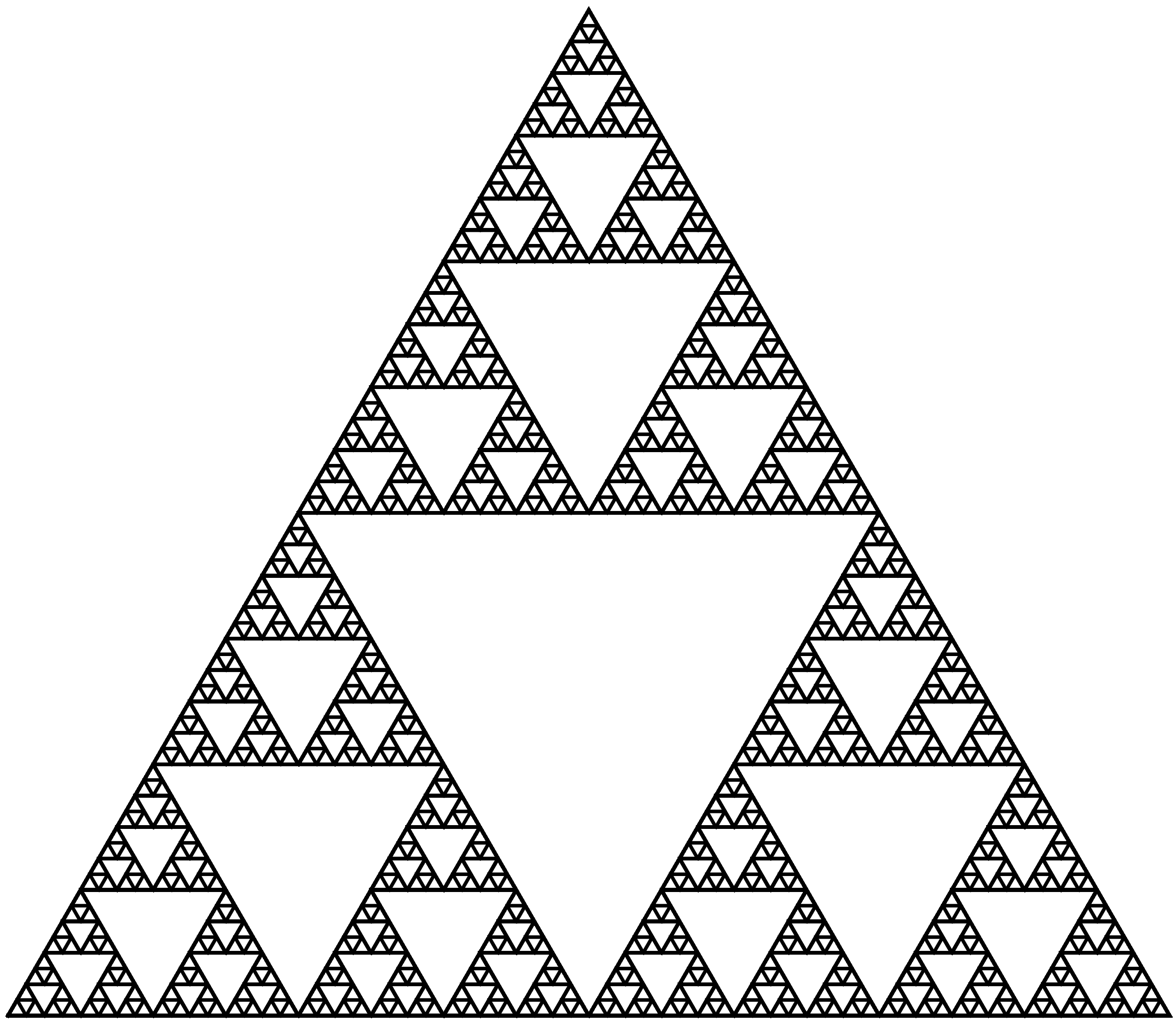

Figure 23: Sierpinski triangle.

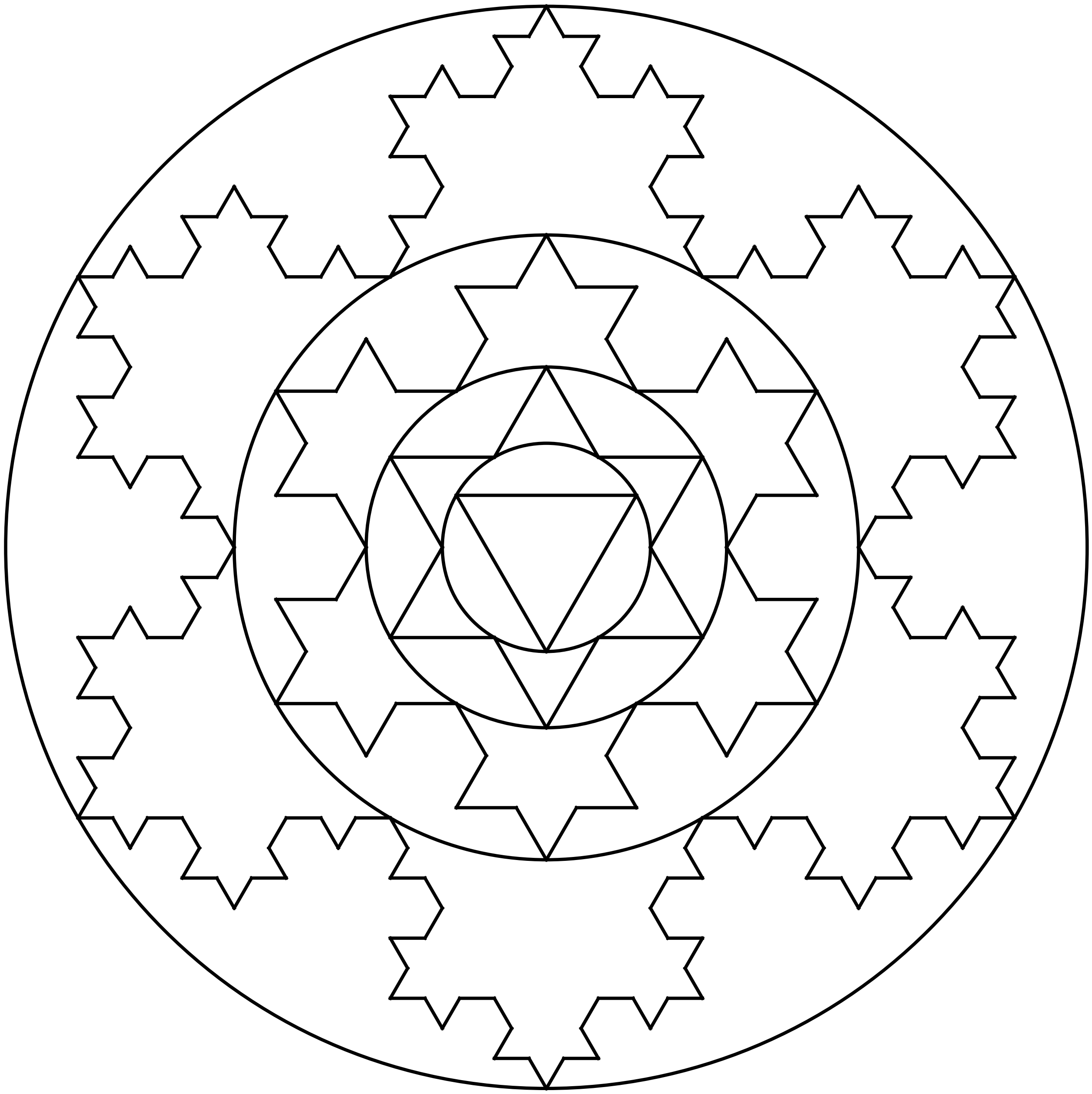

Figure 24: Nested Koch snowflakes and circles

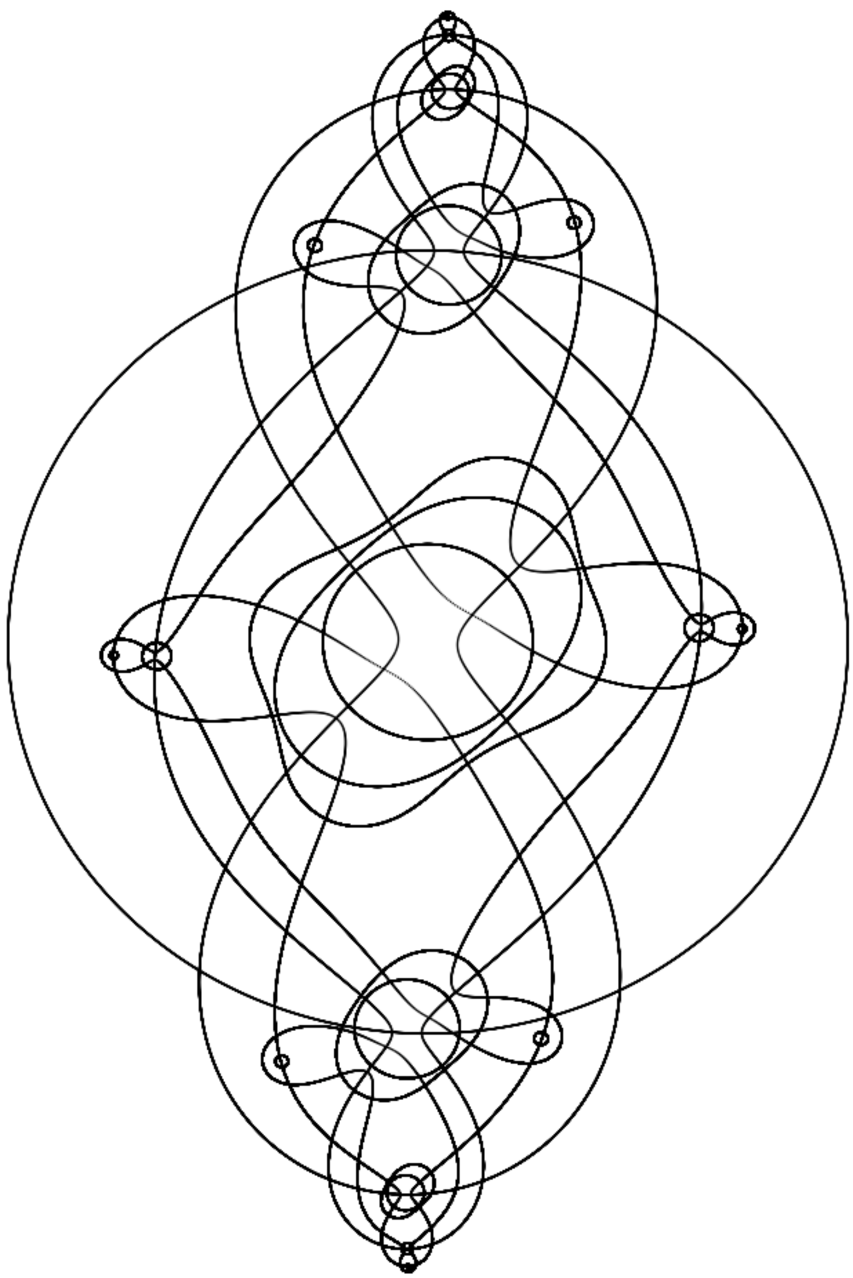

Figure 25: Inverse images of circles under a quadratic map.

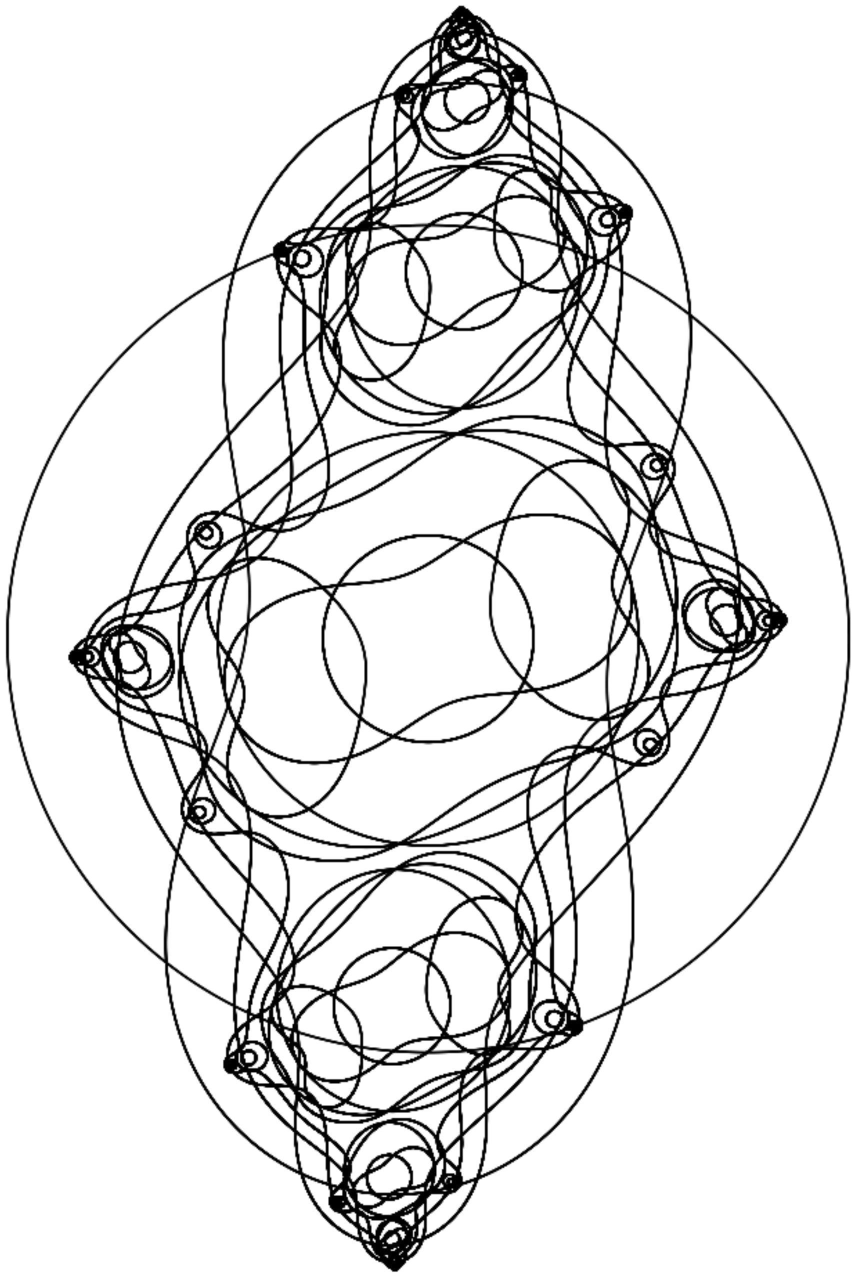

Figure 26: Inverse images of circles under a quadratic map.

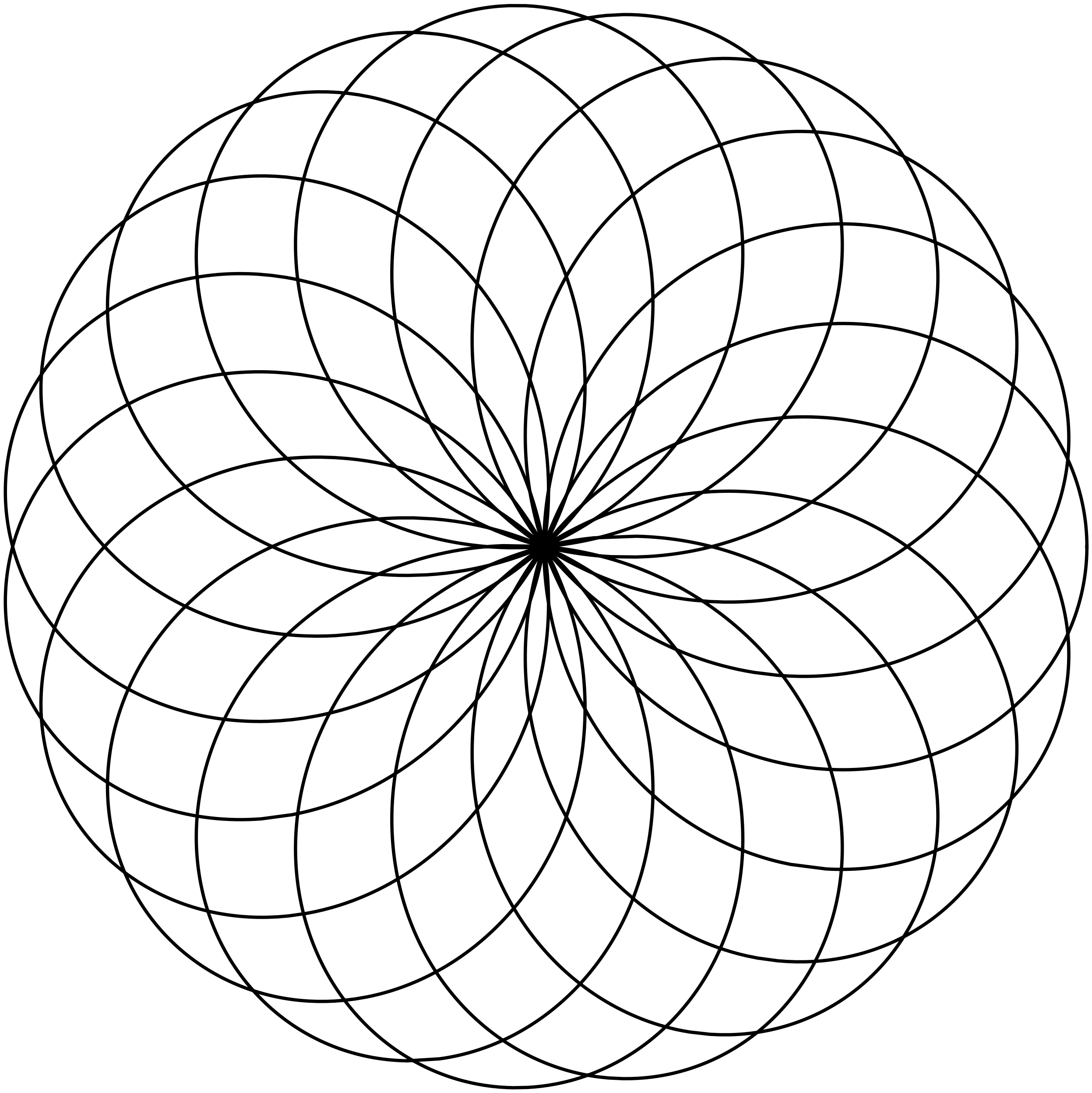

Figure 27: Hypotrochoid.

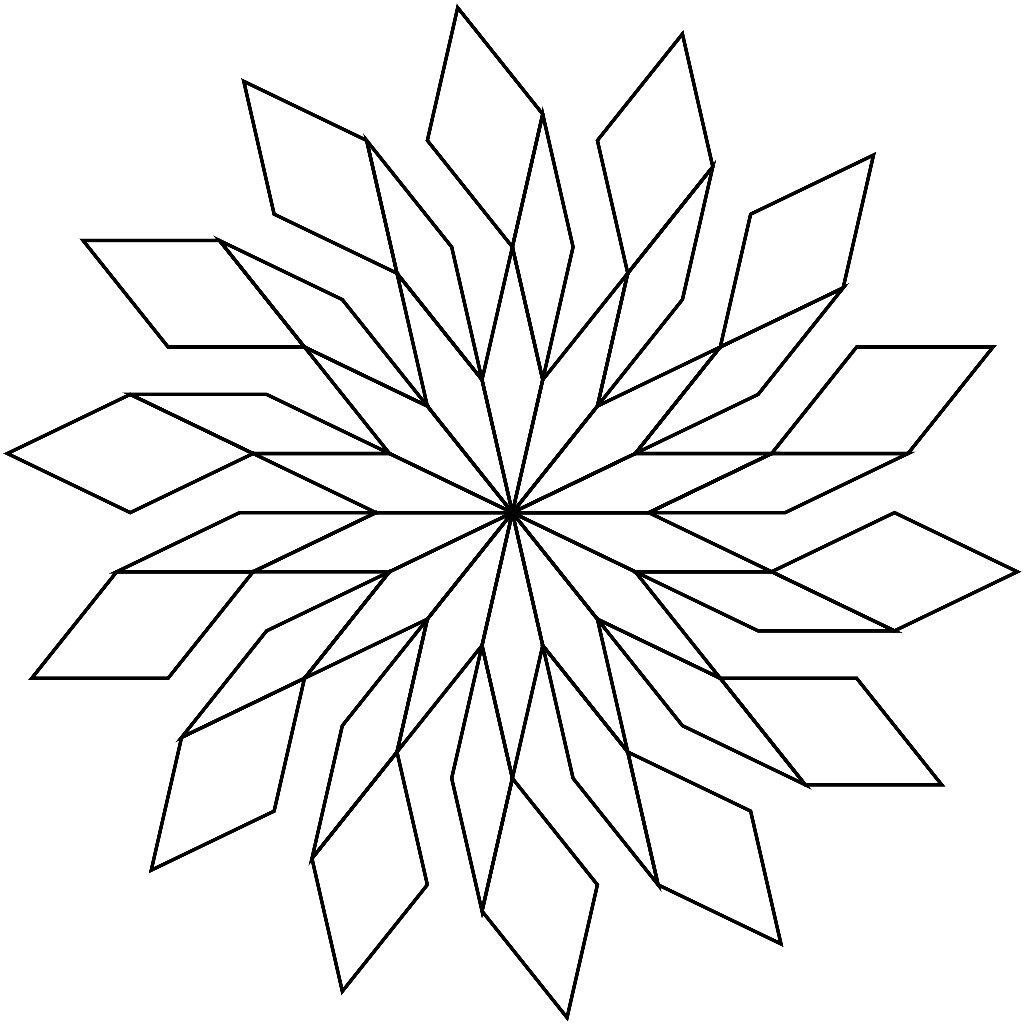

Figure 28: Aperiodic symmetric tiling # 1

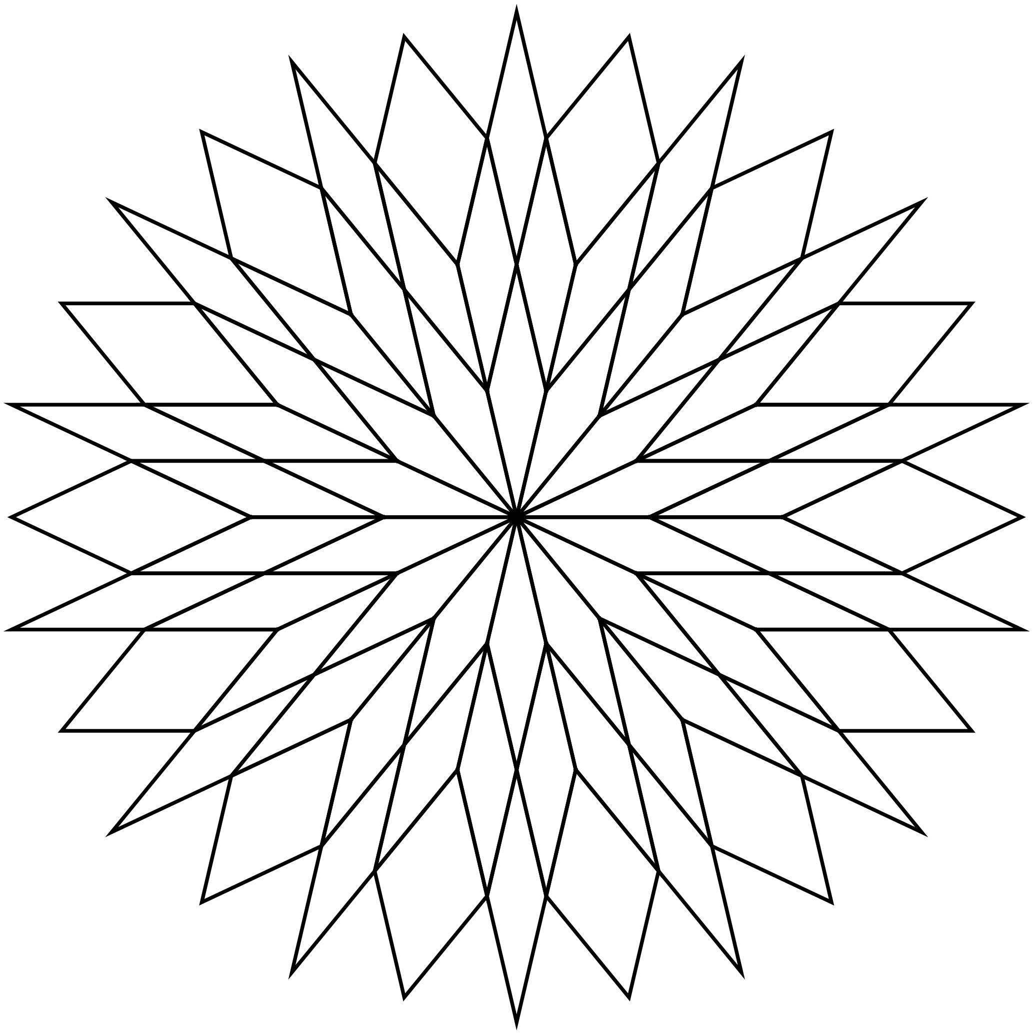

Figure 29: Aperiodic symmetric tiling # 2

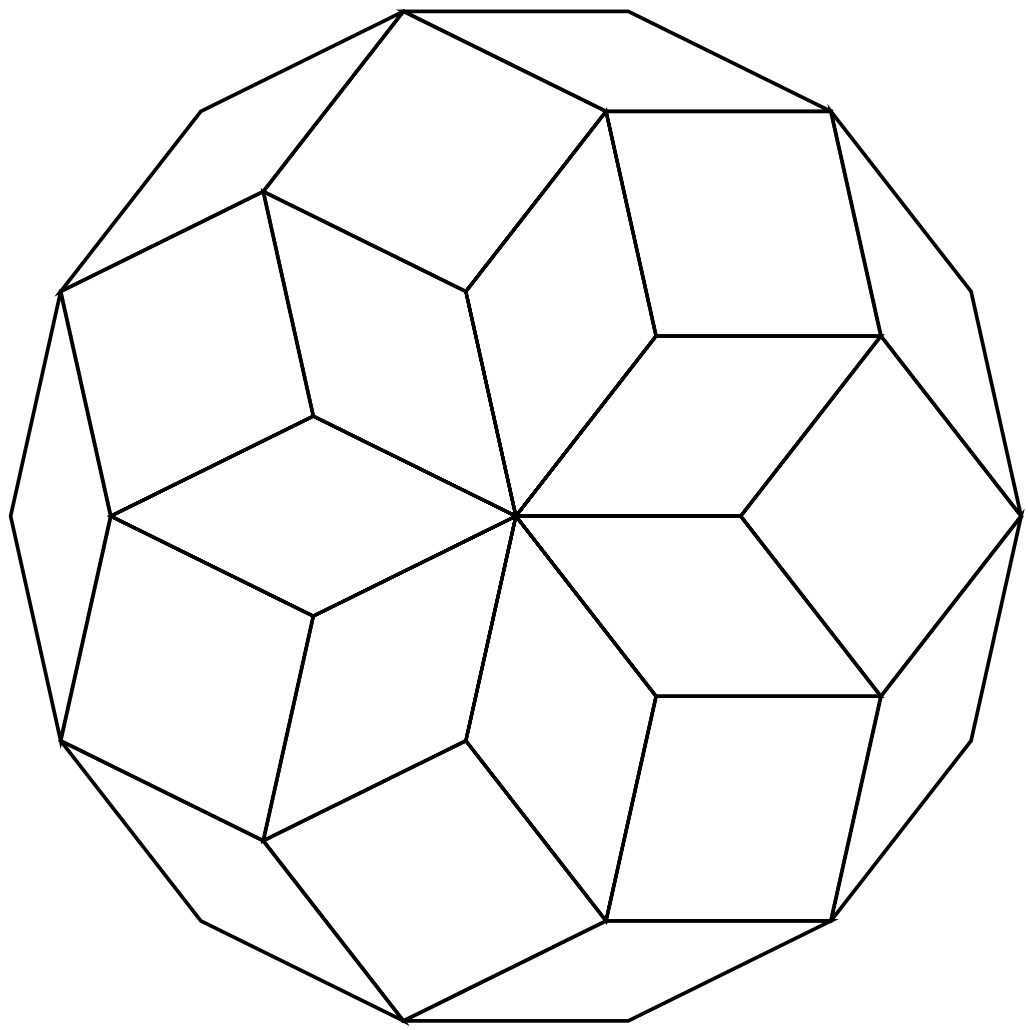

Figure 30: Aperiodic symmetric tiling # 3

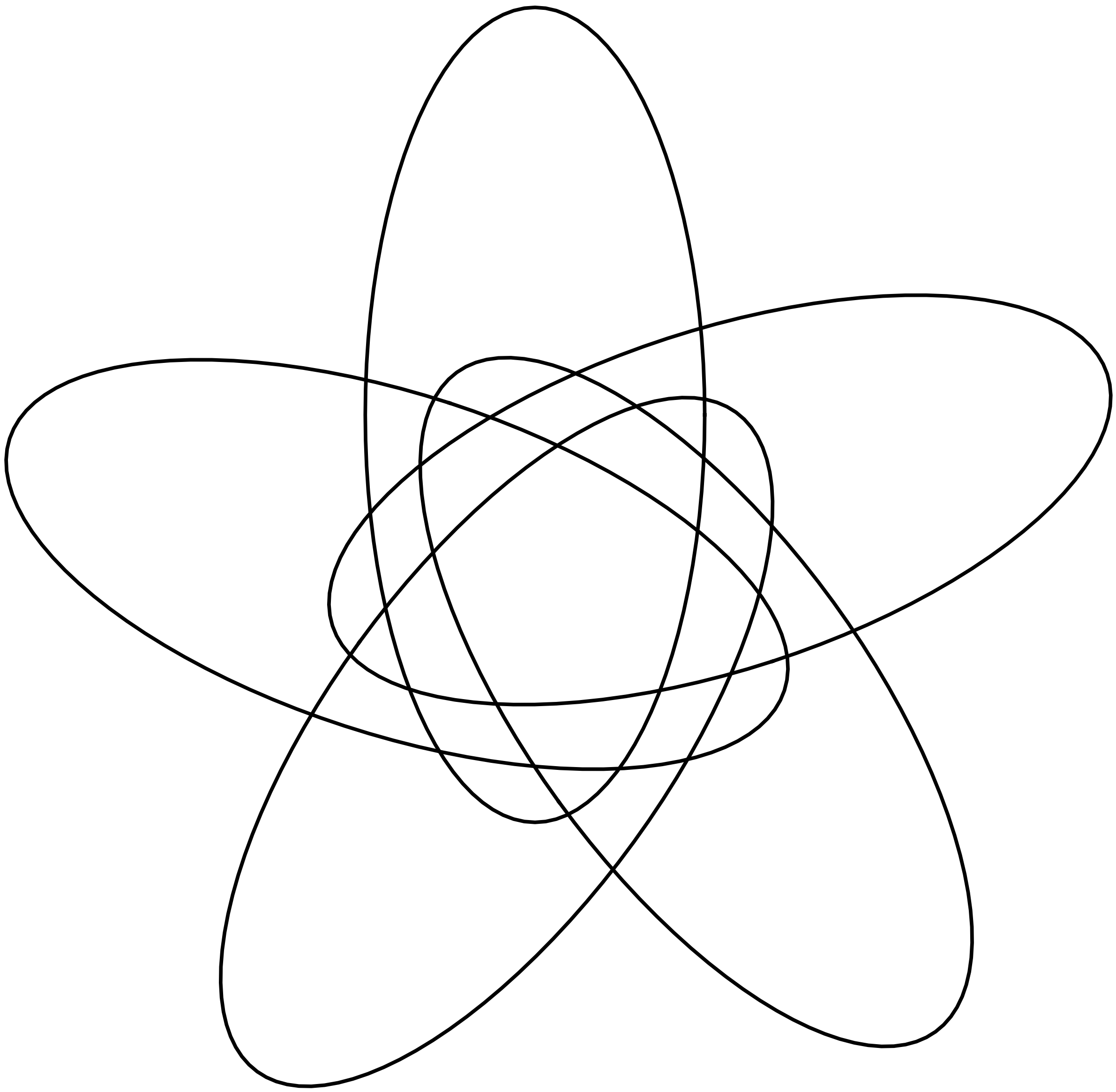

Figure 31: Five set elliptical Venn diagram.

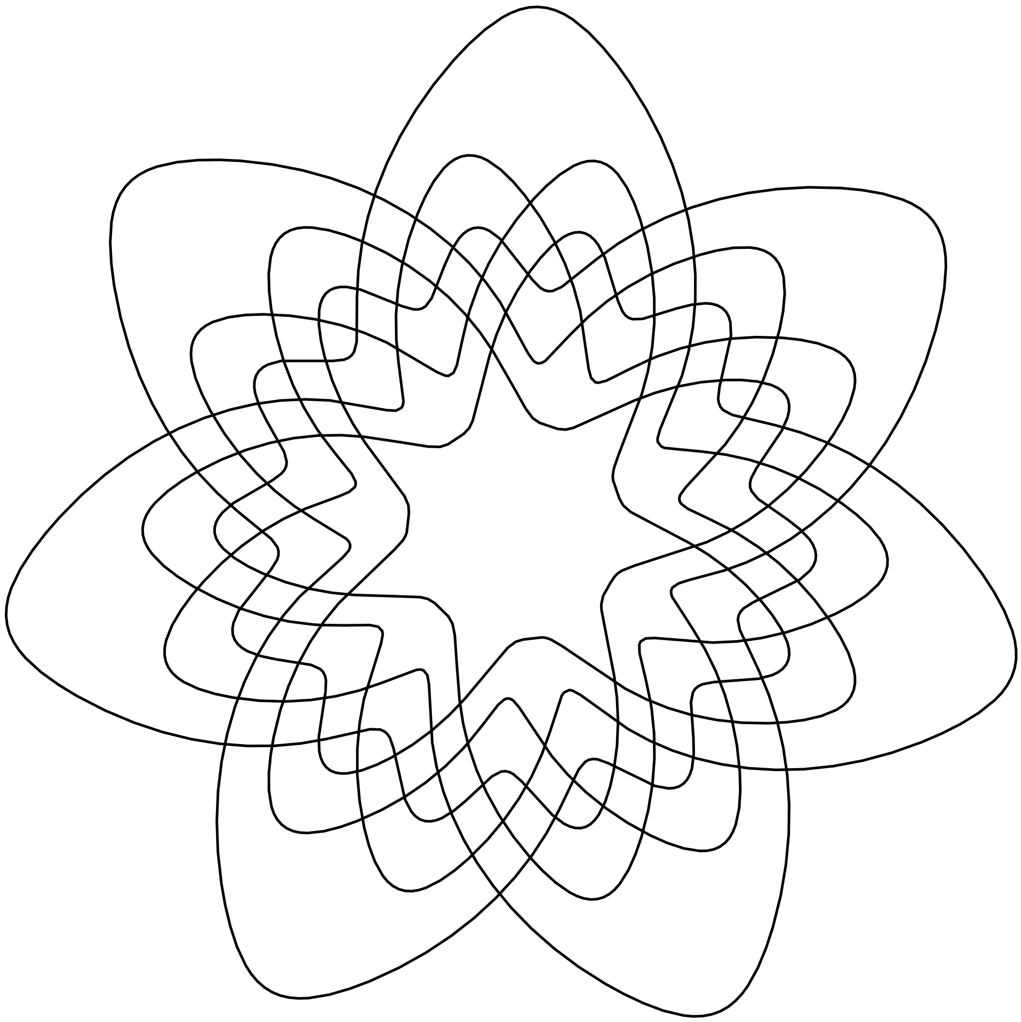

Figure 32: Seven set Venn diagram.

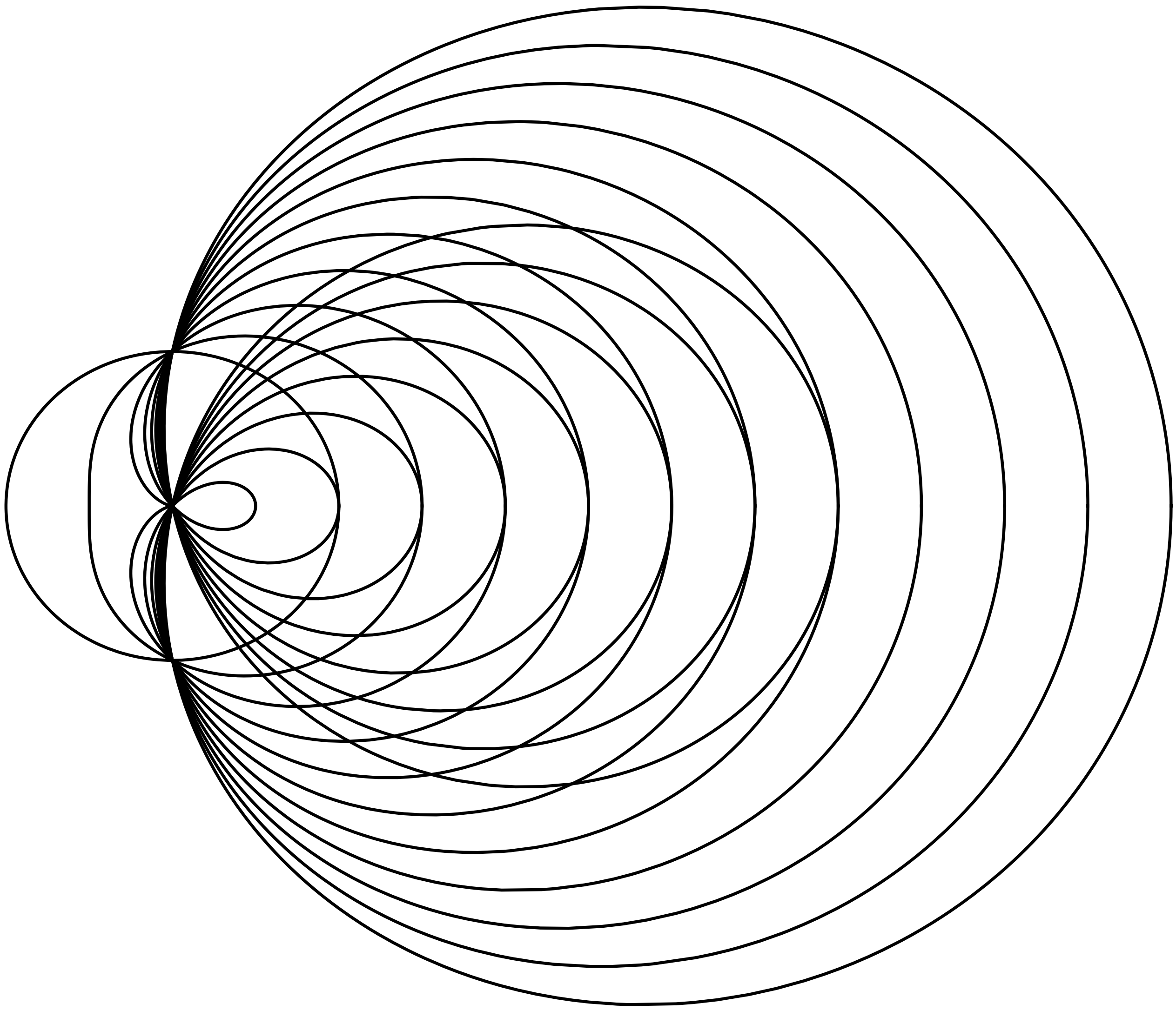

Figure 33: Limacons

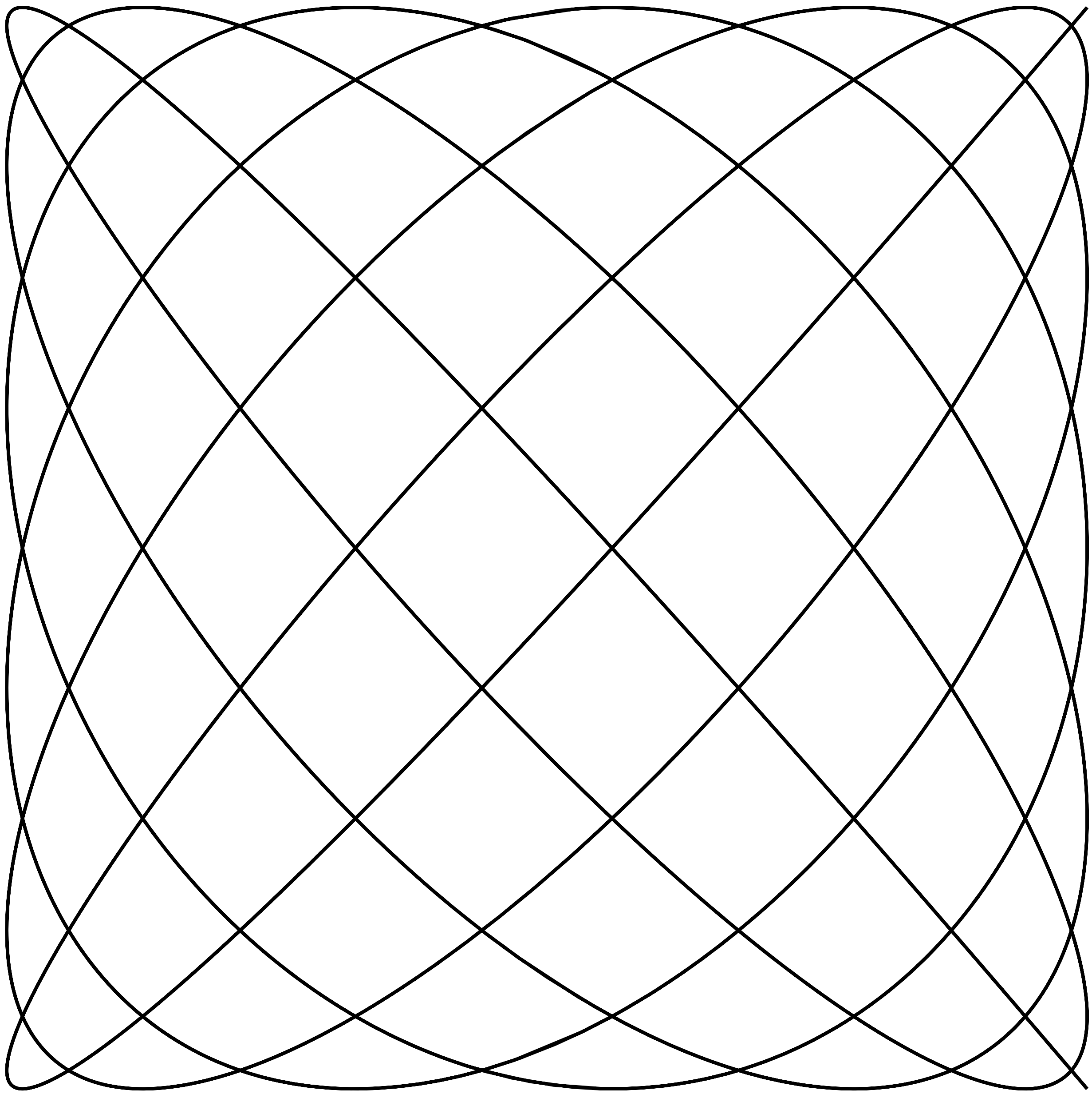

Figure 34: Lissajous ‘pillow’.

Figure Notes

1. Schlegel projection of an icosahedron. In this version a face is selected, and the polyhedron is reflected so that the center of the face points in the $(0, 0, 1)$ direction. The vertices are then scaled so they all lie on the unit sphere. Then the vertices are stereographically projected. The edges are then added in the plane.

2. Schlegel projection of the hypercube.

3. Orthogonal projection of the hypercube.

4. Orthogonal projection of the four-dimensional cross-polytope. The 3-D cross polytope is the octahedron; the cross-polytopes are dual to the n-cubes.

5. Schlegel projection of the twenty-four cell. This is a self-dual regular four-dimensional polytope that does not have a regular 3D analog, although in some ways it is like a 4D version of the truncated cube (cuboctahedron).

6. Orthogonal projection of the twenty-four cell.

7. Orthogonal projection of the 120-cell, the 4D analog of the dodecahedron. It is a regular polytope.

8. Orthogonal projection of the 600-cell, the 4D analog of the icosahedron. It is a regular polytope, dual to the 120-cell.

9. A tiling derived from a projection of the small rhombicuboctahedron.

10. Pursuit curve. This is what happens when we have n points, equally spaced on a circle, and each point moves towards the next (in clockwise order in this illustration, with $n = 5$). In the limit as the stepsize becomes zero, we get the pursuit curves which are solutions to the differential equations $\frac{d\vec{x}_n}{dt} = \vec{x}_{n+1} - \vec{x}_n$.

11. Apollonian circle packing. The curvature of a circle is the inverse of its radius. If four mutally tangent circles with integral curvature are chosen, further choices of circles tangent to three others (an Apollonian packing) will also have integral curvature. Determining the possible distributions of curvatures is a difficult problem.

12. Tangent conics to five circles. In 1848 J. Steiner posed the question of how many complex conics were tangent to five given conics. He thought the number should be 7776, but in 1864 M. Chasles showed the correct number was 3264. In 1984 W. Fulton asked if all 3264 conics could be real; he came up with a proof that the answer was yes, but did not publish it. In 1997 F. Ronga, A. Tognoli, and T. Vust published a complete proof which uses very similar ideas to Fulton's. The idea is to deform a degenerate configuration of 5 lines into five hyperbolae; conics that were tangent to a line deform into multiple conics tangent to the hyperbolae. This made me curious if there could

be 3264 real conics to five circles. I still don't know the answer, but this picture of some of the real tangent conics to five circles on a regular pentagon might help give some intuition...

13. Geodesics in the unit disk model of hyperbolic space. This is an approximation to a tiling by pentagons in which four pentagons meet at each vertex.

14. Part of the hyperbolic plane tiled with stars. In hyperbolic space, each of these stars is the same size and the edges are "straight" - i.e. geodesics.

15. The Gröbner fan of the ideal $< x^5 - y^4, y^5 - z^4, z^5 - x^4 >$.

16. Gröbner fan of the ideal $< x^3 - y^2, y^3 - z^2, z^3 - x^2 >$

17. Gröbner fan of the vortex problem ideal defined by

$$-6xyz + xy + xz + 6yz - y^2 - z^2,$$

$$-6xyz + xy + 6xz + yz - x^2 - z^2,$$

$$-6xyz + 6xy + xz + yz - x^2 - y^2$$

    The variables are the squares of the distances between the three vortices. This equations determine the stationary configurations of three equal-strength vortices.

18. An Archimedean spiral ($r = t$) and an exponential spiral ($r = e^{15.73t}$; chosen arbitrarily to make things look nicer).

19. Cobweb diagram for the logistic map $f(x) = 3.9x(1 - x)$. The graphs of the function $f(x)$ and its first two iterates $f(f(x))$ and $f(f(f(x)))$ are plotted.

20. Cobweb diagram for the logistic map $f(x) = 3.9605x(1-x)$. The graphs of the function $f(x)$, $f(f(x))$ and $f(f(f(f(x))))$ are plotted. This parameter for the logistic map is sitting in a small period-4 window.

21. Equipotential lines for the equal-mass three-body problem. The levels are not equally spaced in value.

22. Orbits in the planar circular restricted three-body problem with $\mu = .5$ (equal mass primaries).

23. Sierpinski triangle. One of the first fractal structures considered, it and similar fractals are important in the topological classification of continua. It has Hausdorff dimension $\log(3)/\log(2) \approx 1.585$.

24. Nested circles and Koch snowflakes (finite iterates). The Koch snowflake is one of the first fractals ever constructed (1906).

25. Inverse images of two circles, radii 1 and 1/4, under a quadratic map. Image is rotated 90 degrees for a better aspect ratio on the page (so the imaginary axis is horizontal). The map is $z \to z^2 - .99 + 0.1I$.

26. Inverse images of three circles, radii 1, 1/2, and 1/4, under a quadratic map. Image is rotated 90 degrees for a better aspect ratio on the page (so the imaginary axis is horizontal). The map is $z \to z^2 - 0.8 + .156I$.

27. Hypotrochoid - ? $(x(t), y(t)) = (8cos(t) + 8cos(17t/2), 8sin(t) - 8sin(17t/2))$.

28. Rotationally symmetric arrangement of parallelogram tiles. The most acute angles in the three types of tiles in this and the next three figures have angles $\pi/7$, $3\pi/14$, and $2\pi/7$.

29. Another rotationally symmetric arrangement of parallelogram tiles.

30. Another rotationally symmetric arrangement of parallelogram tiles.

31. Symmetric Venn diagram for 5 sets represented by ellipses. Discovered by Branko Grunbaum in 1975.

32. "Adelaide". A beautiful 7-set symmetric Venn diagram discovered independently by Branko Grunbaum and Anthony Edwards.

33. Limacons $r = 1 + q\cos(t)$ with $q \in [0, 5]$.

34. Lissajous curve $x = \cos(12t)$, $y = \sin(13t)$.