

#### Relational Calculus

Chapter 4, Part B

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#### Relational Calculus

- Comes in two flavors: <u>Tuple relational calculus</u> (TRC) and <u>Domain relational calculus</u> (DRC).
- Calculus has variables, constants, comparison ops, logical connectives and quantifiers.
  - TRC: Variables range over (i.e., get bound to) tuples.
  - <u>DRC</u>: Variables range over *domain elements* (= field values).
  - Both TRC and DRC are simple subsets of first-order logic.
- Expressions in the calculus are called *formulas*. An answer tuple is essentially an assignment of constants to variables that make the formula evaluate to *true*.

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# Domain Relational Calculus

- \* Query has the form:  $\langle x1, x2,...,xn \rangle | p(\langle x1, x2,...,xn \rangle) \rangle$
- \* Answer includes all tuples  $\langle x1, x2, ..., xn \rangle$  that make the formula  $p(\langle x1, x2, ..., xn \rangle)$  be true.
- Eormula is recursively defined, starting with simple atomic formulas (getting tuples from relations or making comparisons of values), and building bigger and better formulas using the logical connectives.

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#### DRC Formulas

- \* Atomic formula:
  - $\langle x1, x2, ..., xn \rangle \in Rname$ , or  $X \circ p$  Y, or  $X \circ p$  constant
  - op is one of  $\langle , \rangle = \langle , \geq , \neq \rangle$
- \* Formula:
  - an atomic formula, or
  - $\neg p, p \land q, p \lor q$ , where p and q are formulas, or
  - $\exists X(p(X))$ , where variable X is *free* in p(X), or
  - $\forall X(p(X))$ , where variable X is *free* in p(X)
- ❖ The use of quantifiers  $\exists X$  and  $\forall X$  is said to <u>bind</u> X.
  - A variable that is not bound is free.

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#### Free and Bound Variables



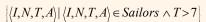
- **❖** The use of quantifiers  $\exists X$  and  $\forall X$  in a formula is said to *bind* X.
  - A variable that is **not bound** is **free**.
- \* Let us revisit the definition of a query:

$$\langle x1, x2, ..., xn \rangle | p(\langle x1, x2, ..., xn \rangle) \rangle$$

There is an important restriction: the variables x1, ..., xn that appear to the left of `|' must be the *only* free variables in the formula p(...).

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# Find all sailors with a rating above



- ❖ The condition  $\langle I, N, T, A \rangle$  ∈ *Sailors* ensures that the domain variables *I*, *N*, *T* and *A* are bound to fields of the same Sailors tuple.
- \* The term  $\langle I, N, T, A \rangle$  to the left of `|' (which should be read as *such that*) says that every tuple  $\langle I, N, T, A \rangle$  that satisfies T > 7 is in the answer.
- \* Modify this query to answer:
  - Find sailors who are older than 18 or have a rating under 9, and are called 'Joe'.

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#### Find sailors rated > 7 who've reserved boat #103

$$\langle \langle I, N, T, A \rangle | \langle I, N, T, A \rangle \in Sailors \land T > 7 \land$$
  
 $\exists Ir, Br, D | \langle Ir, Br, D \rangle \in Reserves \land Ir = I \land Br = 103 |$ 

- \* We have used  $\exists Ir, Br, D (...)$  as a shorthand for  $\exists Ir (\exists Br (\exists D (...)))$
- ❖ Note the use of ∃ to find a tuple in Reserves that `joins with' the Sailors tuple under consideration.

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#### Find sailors rated > 7 who've reserved a red boo

$$\langle I, N, T, A \rangle | \langle I, N, T, A \rangle \in Sailors \land T > 7 \land$$
  
 $\exists Ir, Br, D \langle Ir, Br, D \rangle \in Reserves \land Ir = I \land$   
 $\exists B, BN, C \langle B, BN, C \rangle \in Boats \land B = Br \land C = 'red' \rangle$ 

- Observe how the parentheses control the scope of each quantifier's binding.
- This may look cumbersome, but with a good user interface, it is very intuitive. (MS Access, QBE)

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### Find sailors who've reserved all boats

$$\langle (I, N, T, A) | \langle I, N, T, A \rangle \in Sailors \land$$

$$\forall B, BN, C \left[ \neg [\langle B, BN, C \rangle \in Boats] \lor$$

$$(\exists Ir, Br, D | \langle Ir, Br, D \rangle \in Reserves \land I = Ir \land Br = B) |$$

\* Find all sailors I such that for each 3-tuple  $\langle B,BN,C\rangle$  either it is not a tuple in Boats or there is a tuple in Reserves showing that sailor I has reserved it.

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### Find sailors who've reserved all boats (again)

$$\langle I, N, T, A \rangle | \langle I, N, T, A \rangle \in Sailors \land$$

$$\forall \langle B, BN, C \rangle \in Boats$$

$$(\exists \langle Ir, Br, D \rangle \in Reserves [I = Ir \land Br = B])$$

- Simpler notation, same query. (Much clearer!)
- \* To find sailors who've reserved all red boats:

..... 
$$(C \neq 'red' \vee \exists \langle Ir, Br, D \rangle \in Reserves(I = Ir \wedge Br = B))$$

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# Unsafe Queries, Expressive Power

It is possible to write syntactically correct calculus queries that have an infinite number of answers! Such queries are called <u>unsafe</u>.

• e.g., 
$$|S| \neg [S \in Sailors]$$

- It is known that every query that can be expressed in relational algebra can be expressed as a safe query in DRC / TRC; the converse is also true.
- <u>Relational Completeness</u>: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.

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## Summary

- Relational calculus is non-operational, and users define queries in terms of what they want, not in terms of how to compute it. (Declarativeness.)
- Algebra and safe calculus have same expressive power, leading to the notion of relational completeness.

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