## Bottom-Up Parsing Algorithms

- LR(k) parsing
  - L: scan input Left to right
  - R: produce Rightmost derivation
  - k tokens of lookahead
- LR(0)
  - zero tokens of look-ahead
- SLR
  - Simple LR: like LR(0), but uses FOLLOW sets to build more "precise" parsing tables
  - LR(0) is a toy, so we focus on SLR
- Reading: Section 4.7

## Problem: when to shift, when to reduce?

- Recall our favorite grammar:

  E → T + E | T

  T → int * T | int | (E)
- The step

  T * int + int → int  * int + int

  is not part of any rightmost derivation
- Hence, reducing first int to T was a mistake
- *How to know when to reduce and when to shift?*

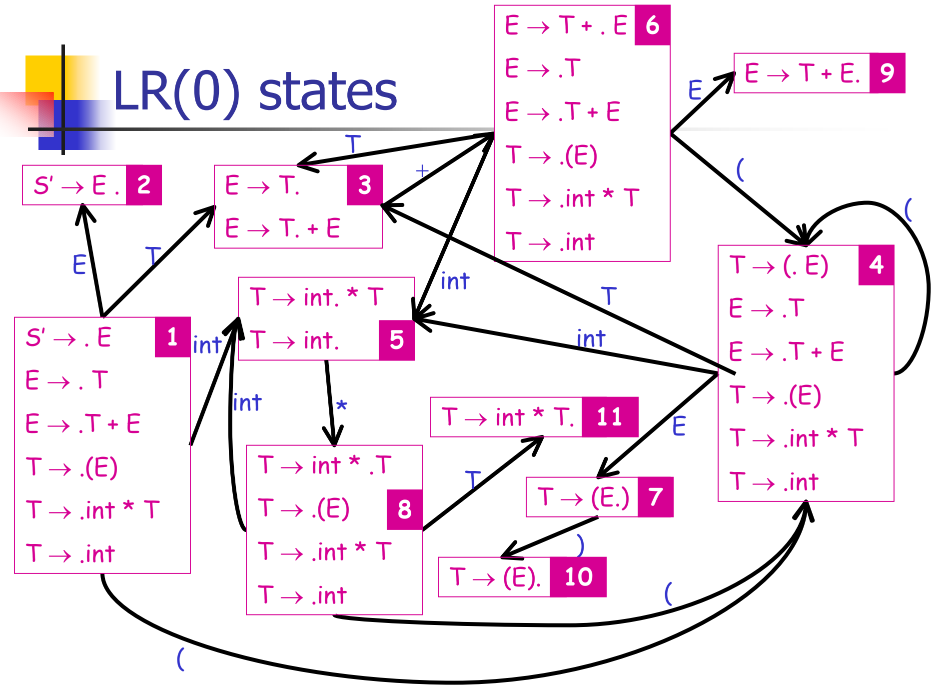
## What we need for LR parsing

- LR(0) states
  - describe states in which the parser can be
  - Note: LR(0) states are used by both LR(0) and SLR parsers
- Parsing tables
  - transitions between LR(0) states,
  - actions to take when transiting:
    - shift, reduce, accept, error
- How to construct LR(0) states?
- How to construct parsing tables?
- How to drive the parser?

## LR(0) state = set of LR(0) items

- An LR(0) item [X $\rightarrow$ $\alpha$ . $\beta$] says that
  - the parser is looking for an X
  - it has an $\alpha$ on top of the stack
  - expects to find input string derived from $\beta$
- Notes:
  - [X $\rightarrow$ $\alpha$ . a$\beta$] means that if a is on the input, it can be shifted (resulting in $\alpha$a . $\beta$). That is:
    - *a* is a correct token to see on the input, and
    - shifting *a* would not "over-shift" (still a viable prefix).
  - [X $\rightarrow\alpha$.] means that we could reduce $\alpha$ to X

# LR(0) states



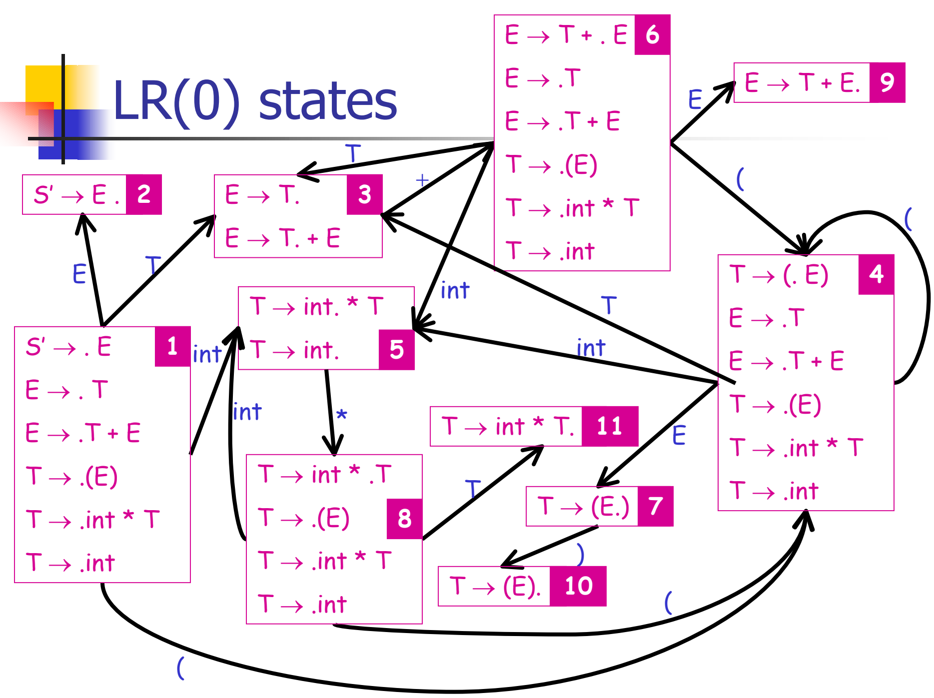

# Naïve SLR Parsing Algorithm

1. Let M be LR(0) state machine for G
   - each state contains a set I of LR(0) items
2. Let $|x_1 \ldots x_n\$$ be initial configuration
3. Repeat until configuration is $S|\$$
   - Let $\alpha|\omega$ be current configuration
   - Run M on current stack $\alpha$
   - If M rejects $\alpha$, report parsing error
   - If M accepts $\alpha$, let a be next input
     - Shift if $[X \to \beta.\ a\ \gamma\ ] \in$ Items
     - Reduce if $[X \to \beta.] \in$ Items and $a \in$ Follow($\alpha$)
       $\ldots\ \beta\ |\ a \ldots \rightarrow \ldots\ |\ X\ a \ldots$
     - Report parsing error if neither applies

# Notes

- If there is a conflict in the last step, grammar is not SLR(k)
- k is the amount of lookahead
  - In practice k = 1

# LR(0) states



# SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | |
| | | |
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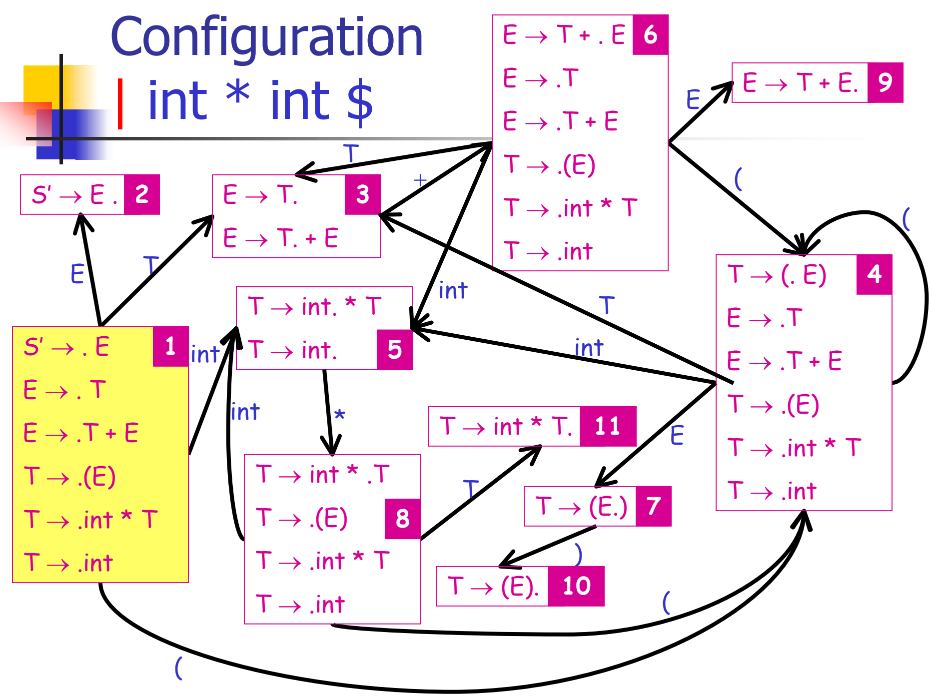
# Configuration

## | int * int $



# SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 | |
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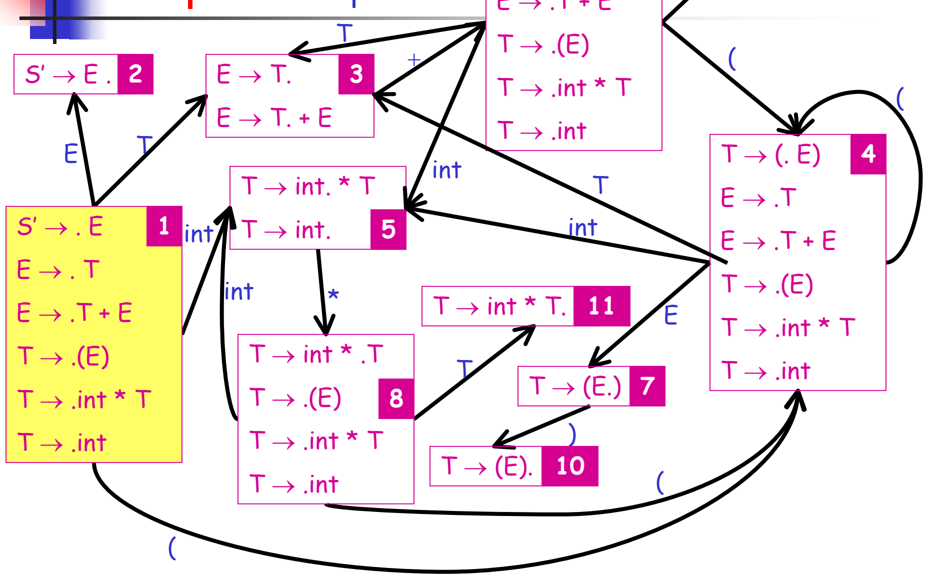
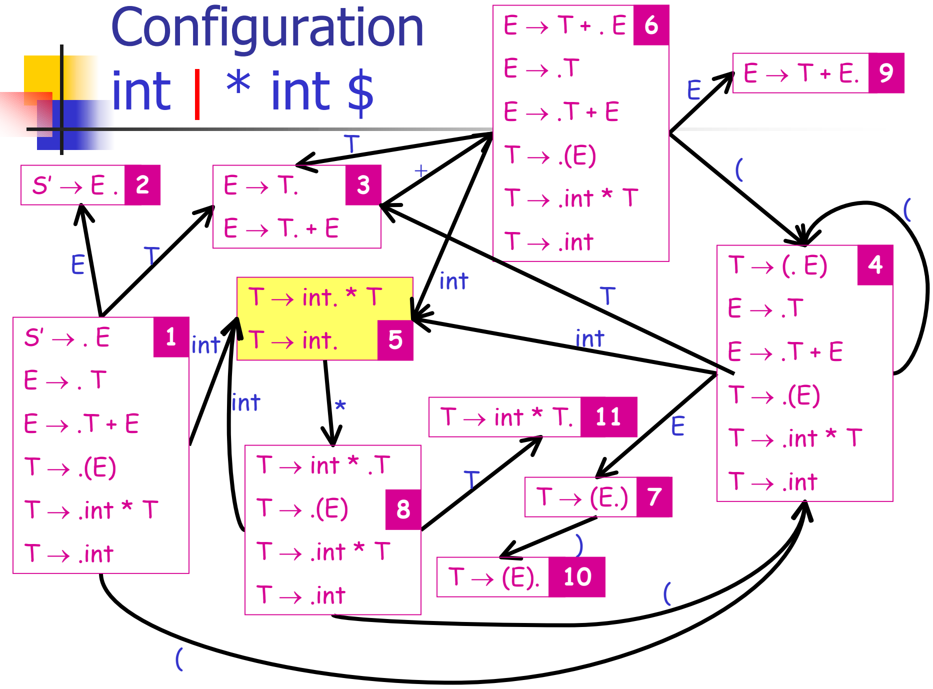
# Configuration
# int | * int $



## Configuration
## int | * int $




## SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | |
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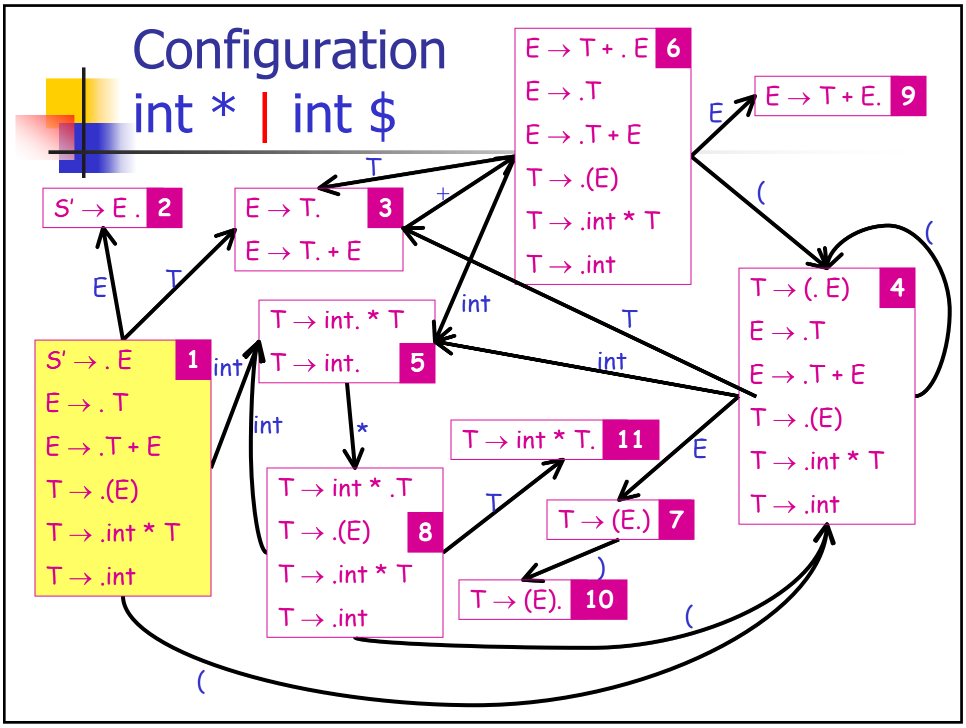
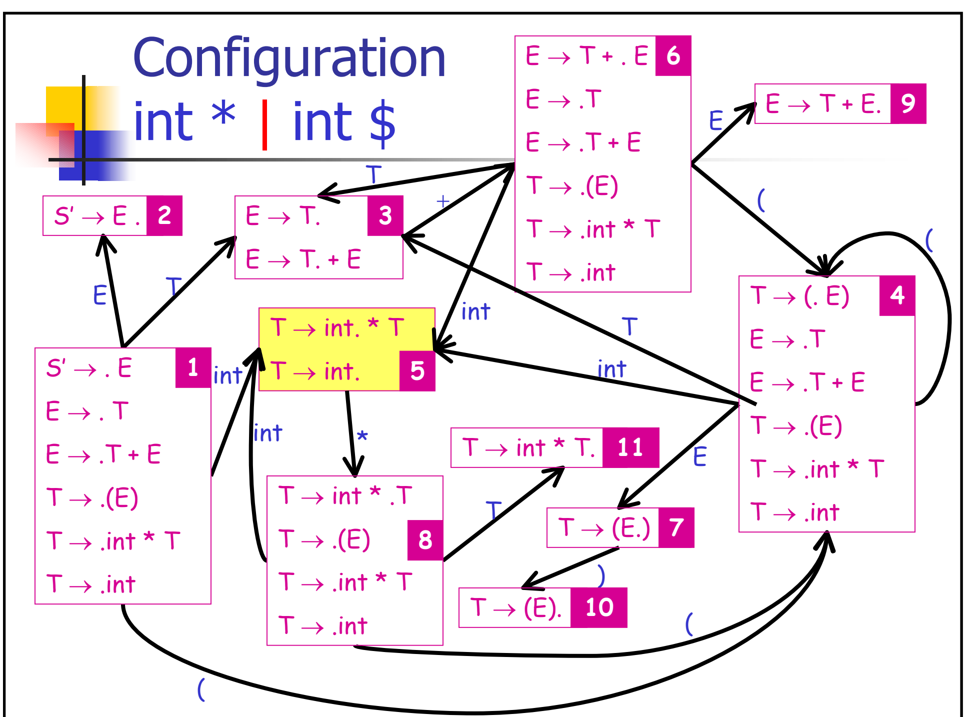

Configuration
int * | int $
S' → E . 2
E → T. 3
E → T. + E
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int. * T
T → int. 5
T → int * .T
T → .(E) 8
T → .int * T
T → .int
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
E
T
int
*
+
(
)


Configuration
int * | int $
S' → E . 2
E → T. 3
E → T. + E
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int. * T
T → int. 5
T → int * .T
T → .(E) 8
T → .int * T
T → .int
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
E
T
int
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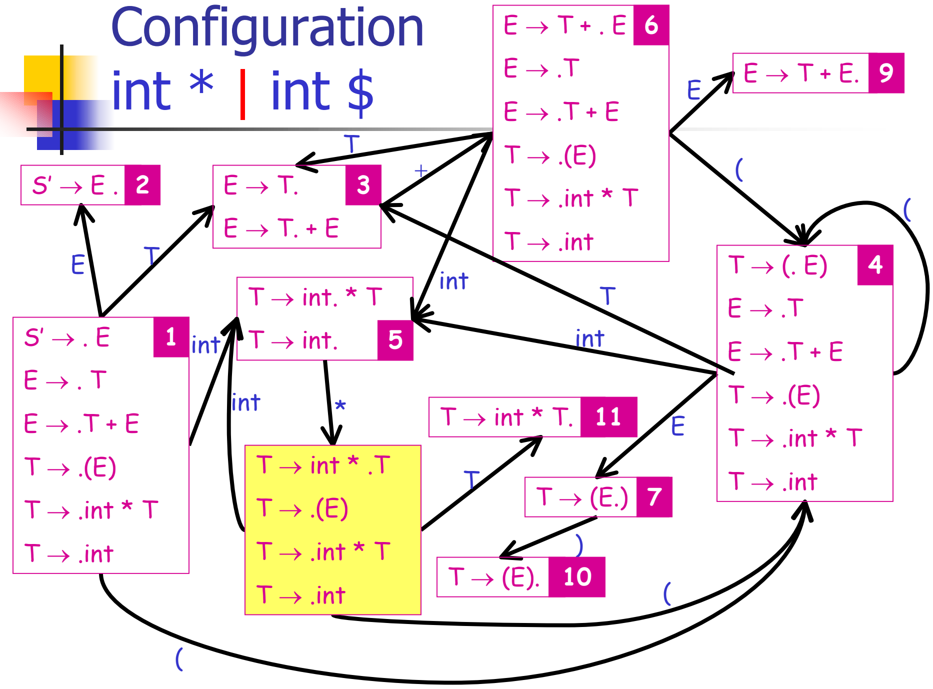
)

# Configuration
# int * | int $




# SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | shift |
| int * int \| $ | 5 | |
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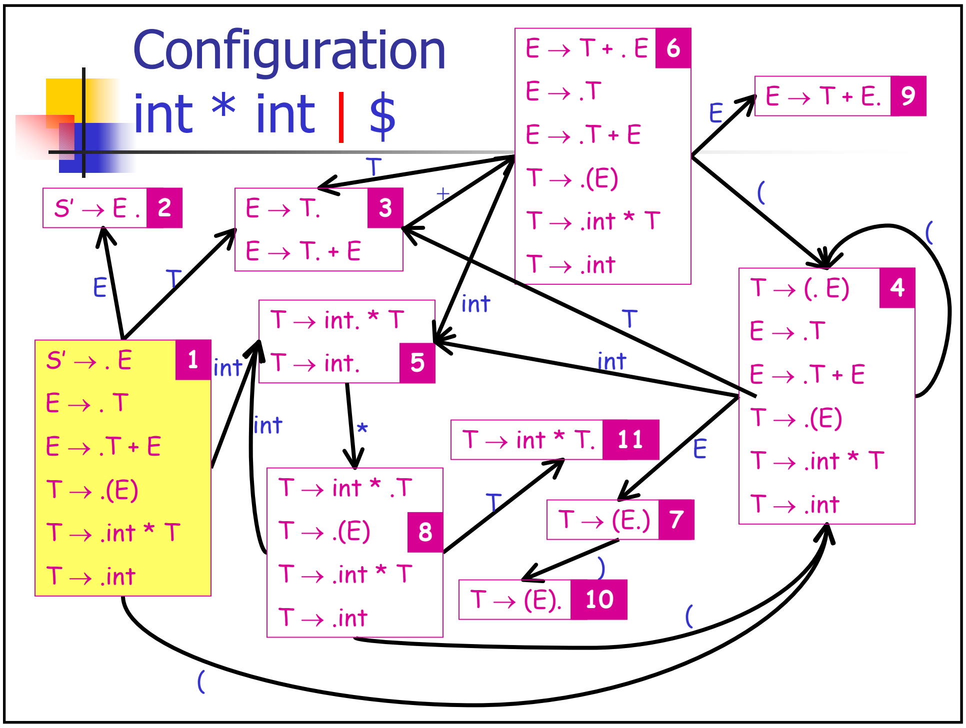
Configuration
int * int | $
S' → E . 2
E → T. 3
E → T. + E
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int. * T
T → int. 5
T → int * .T
T → .(E) 8
T → .int * T
T → .int
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
E
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int
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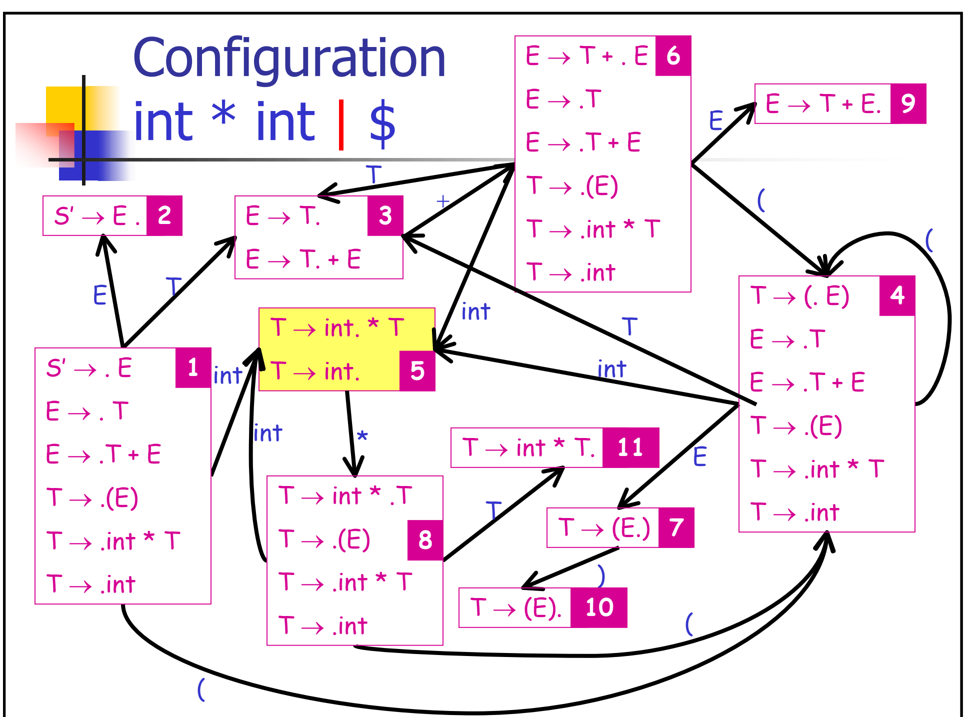
Configuration
int * int | $
S' → E . 2
E → T. 3
E → T. + E
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int. * T
T → int. 5
T → int * .T
T → .(E) 8
T → .int * T
T → .int
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
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E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
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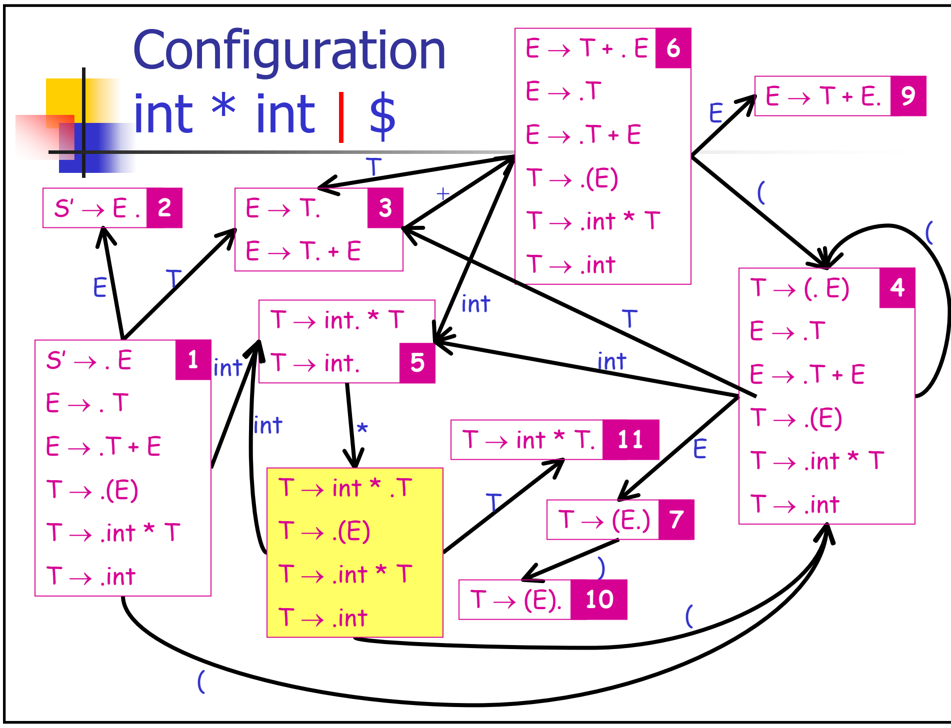
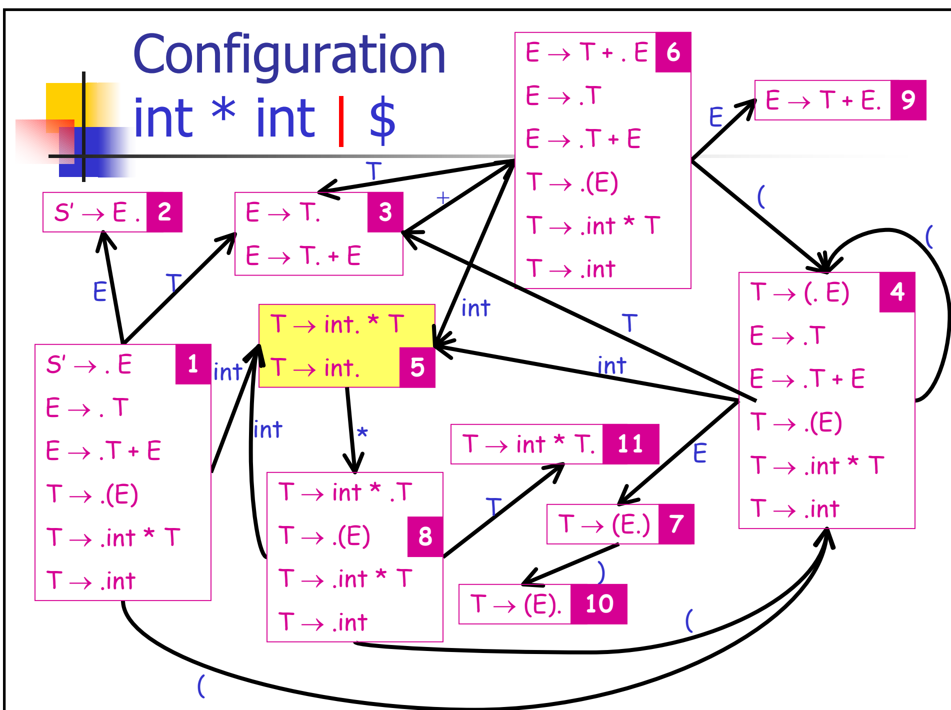
## Configuration
int * int | $




## Configuration
int * int | $



# SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | shift |
| int * int \| $ | 5 $ ∈ Follow(T) | reduce T→int |
| int * \| T $ | 8 | |
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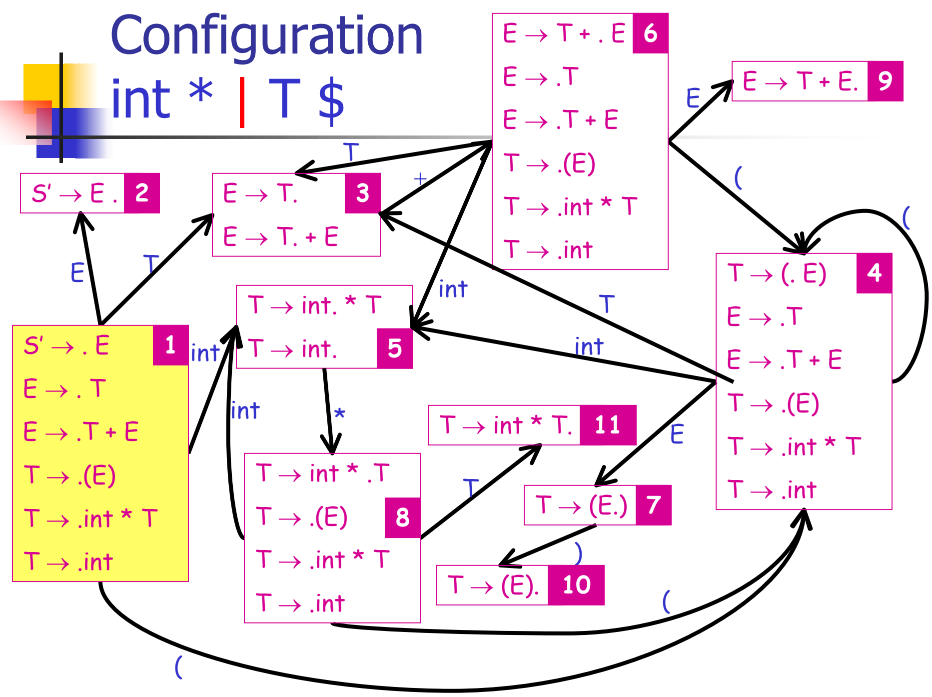
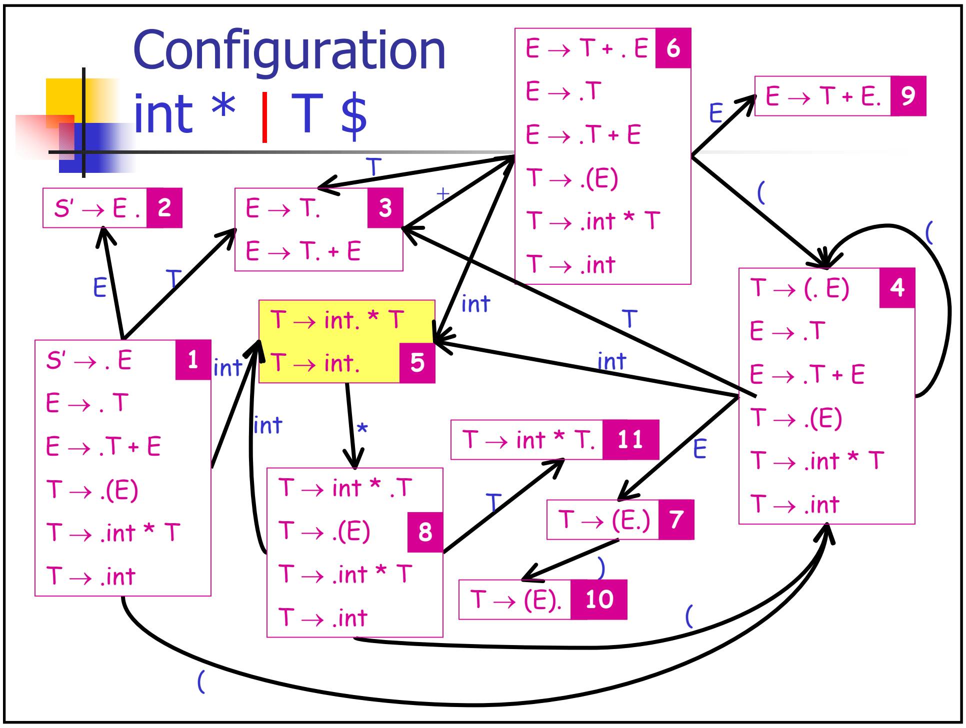
# Configuration int * | T $




Configuration
int * | T $
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
S' → E . 2
E → T. 3
E → T. + E
T → int. * T
T → int. 5
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → int * .T
T → .(E) 8
T → .int * T
T → .int
T → (E.) 7
T → (E). 10
E
T
int
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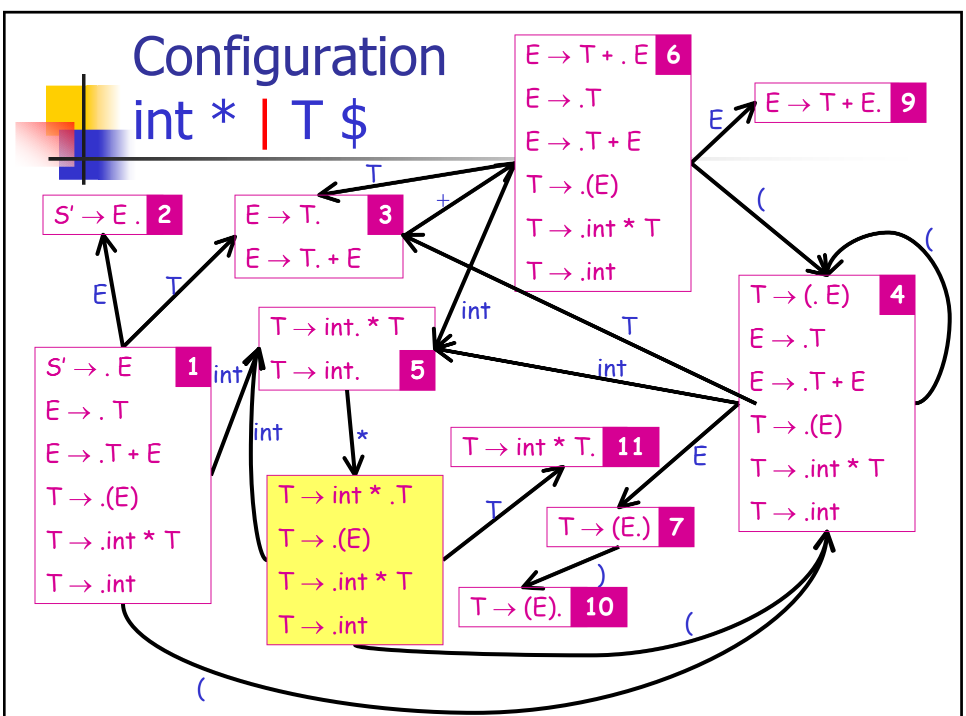
Configuration
int * | T $
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
S' → E . 2
E → T. 3
E → T. + E
T → int. * T
T → int. 5
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → int * .T
T → .(E) 8
T → .int * T
T → .int
T → (E.) 7
T → (E). 10
E
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# SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | shift |
| int * int \| $ | 5 $ ∈ Follow(T) | reduce T→int |
| int * \| T $ | 8 | shift |
| int * T \| $ | 11 | |
| | | |
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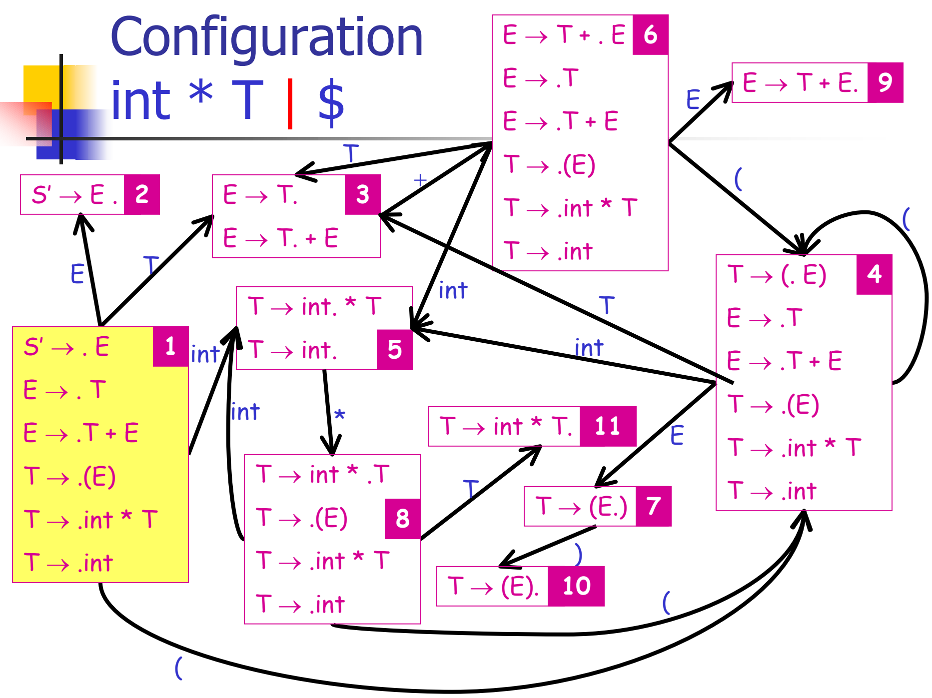
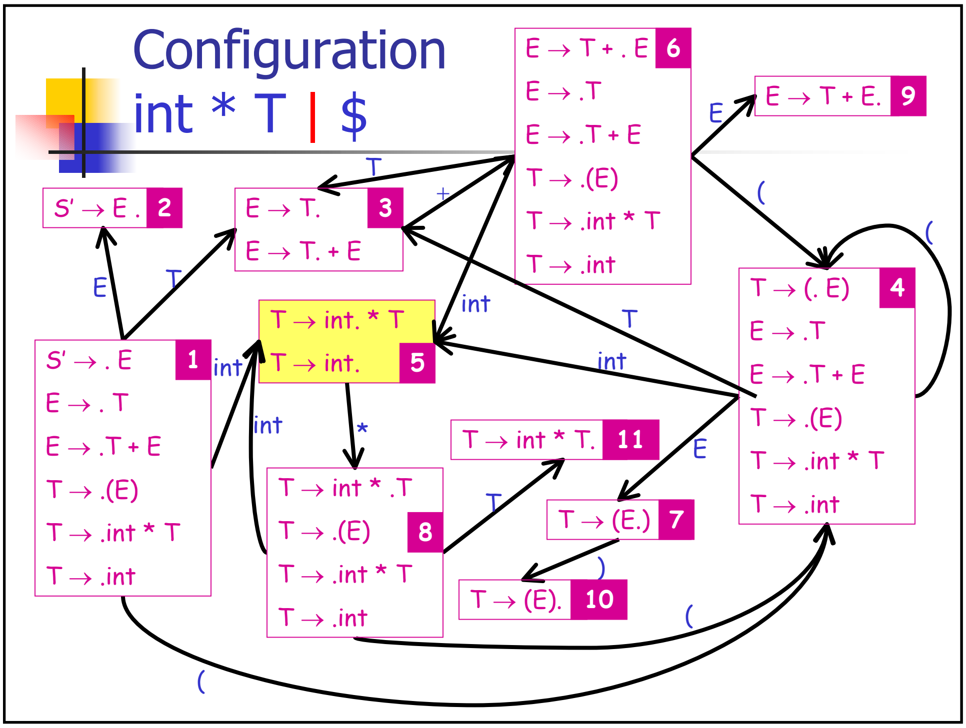
# Configuration
# int * T | $




Configuration
int * T | $
S' → E . 2
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T. 3
E → T. + E
T → int. * T
T → int. 5
T → int * .T
T → .(E) 8
T → .int * T
T → .int
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
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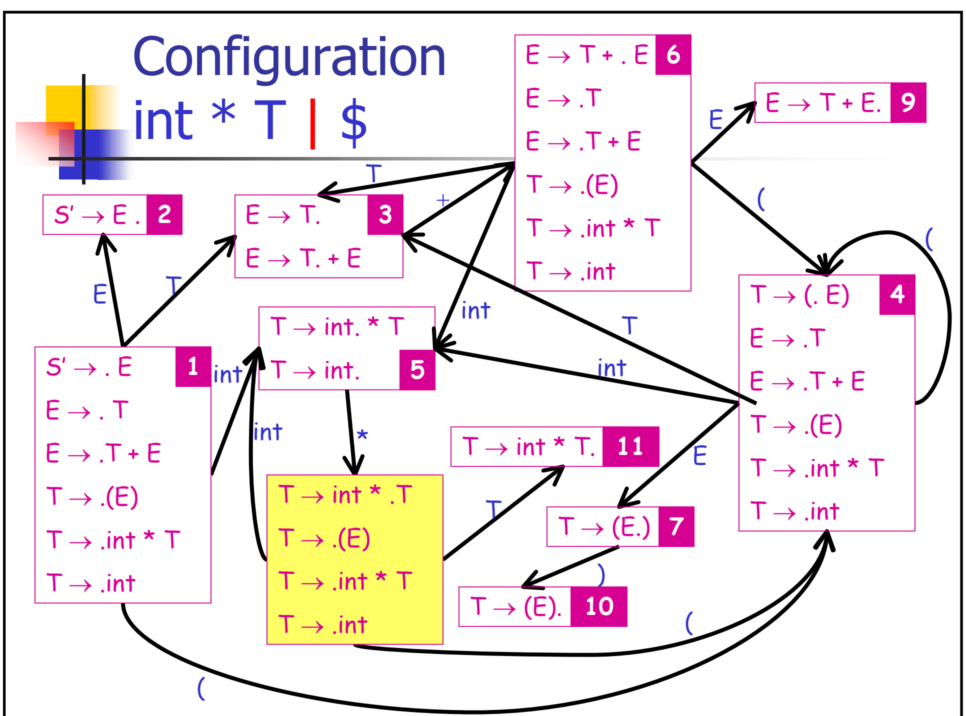
Configuration
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S' → E . 2
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T. 3
E → T. + E
T → int. * T
T → int. 5
T → int * .T
T → .(E) 8
T → .int * T
T → .int
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
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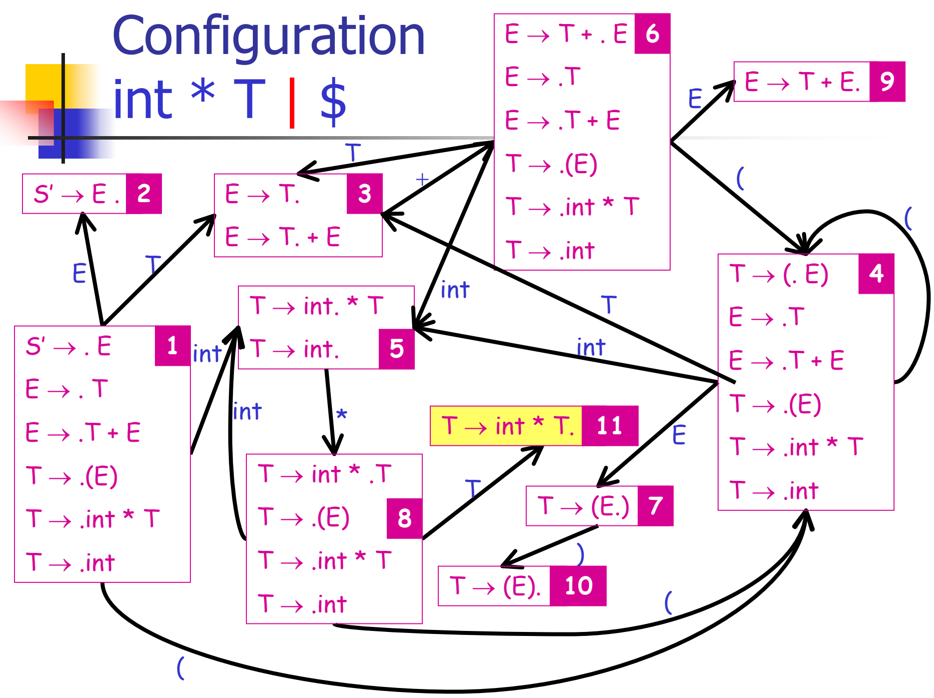
# Configuration int * T | $




# SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | shift |
| int * int \| $ | 5 $ ∈ Follow(T) | reduce T→int |
| int * \| T $ | 8 | shift |
| int * T \| $ | 11 $ ∈ Follow(T) | reduce T→int * T |
| \| T $ | 1 | |
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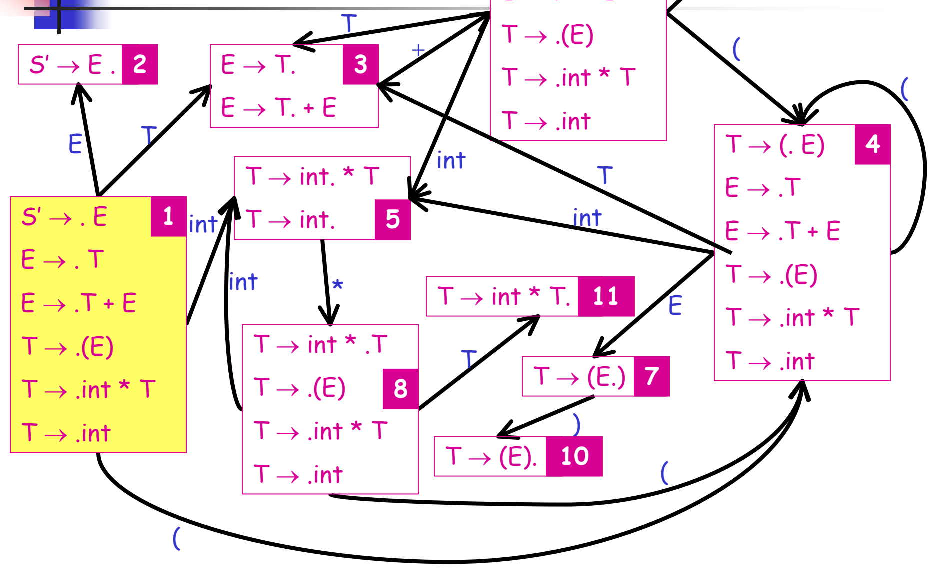
| | | |
| | | |

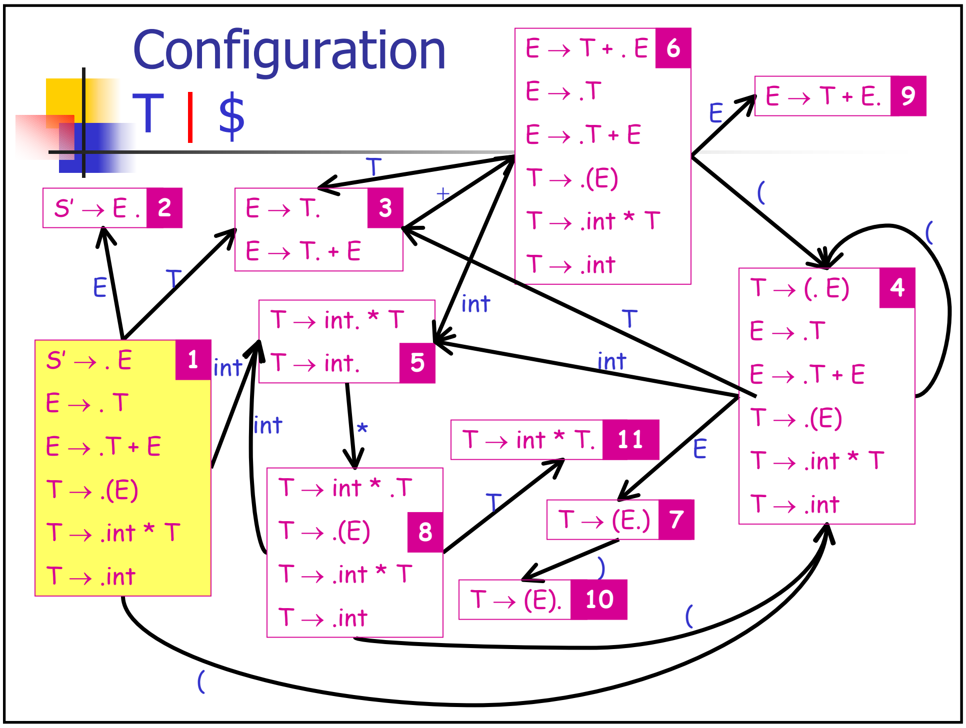
## Configuration

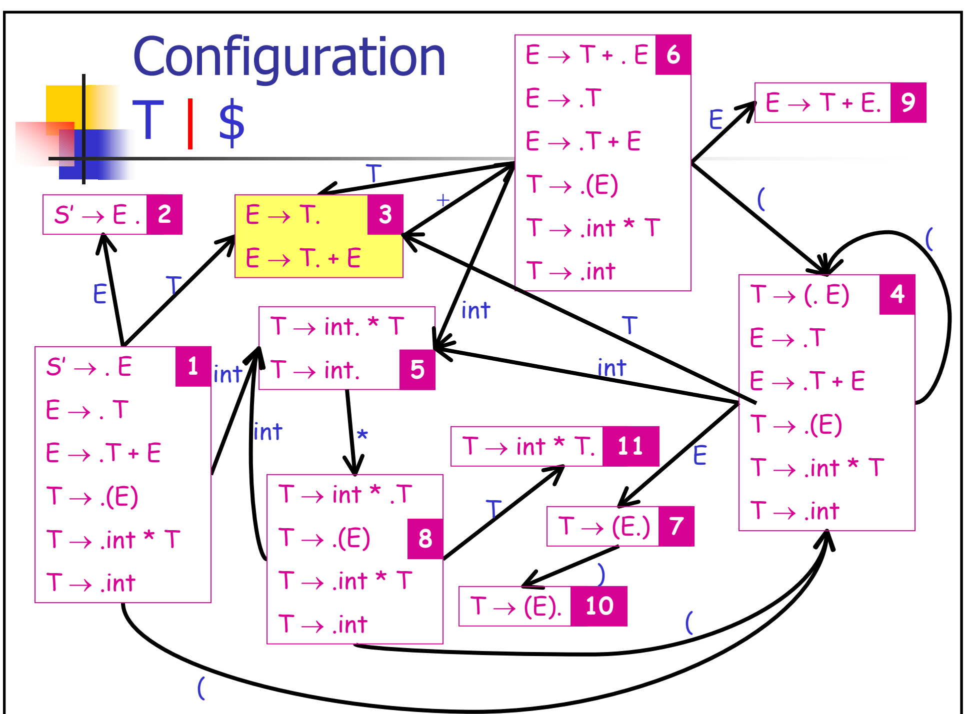
| T $




## SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | shift |
| int * int \| $ | 5 $ ∊ Follow(T) | reduce T→int |
| int * \| T $ | 8 | shift |
| int * T \| $ | 11 $ ∊ Follow(T) | reduce T→int * T |
| \| T $ | 1 | shift |
| T \| $ | 3 | |
| | | |
| | | |


Configuration
T | $
S' → E . 2
E → T. 3
E → T. + E
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int. * T
T → int. 5
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
T → int * .T
T → .(E) 8
T → .int * T
T → .int

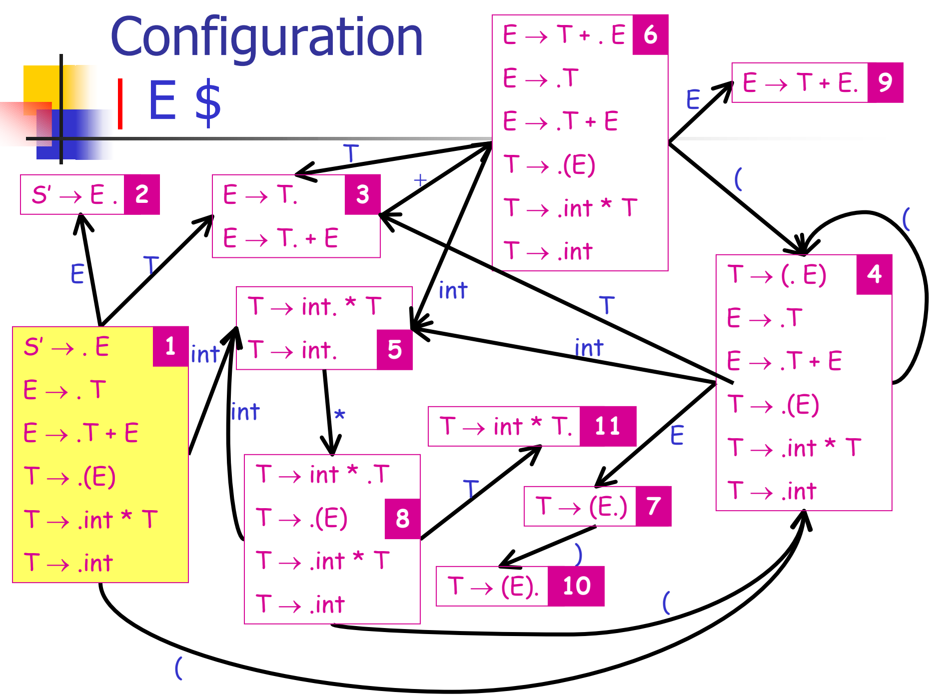

Configuration
T | $
S' → E . 2
E → T. 3
E → T. + E
S' → . E 1
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int. * T
T → int. 5
E → T + . E 6
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
E → T + E. 9
T → (. E) 4
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
T → int * T. 11
T → (E.) 7
T → (E). 10
T → int * .T
T → .(E) 8
T → .int * T
T → .int

# SLR Example

| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | shift |
| int * int \| $ | 5 $ ∊ Follow(T) | reduce T→int |
| int * \| T $ | 8 | shift |
| int * T \| $ | 11 $ ∊ Follow(T) | reduce T→int * T |
| \| T $ | 1 | shift |
| T \| $ | 3 $ ∊ Follow(E) | reduce E→T |
| \| E $ | 1 | |
| | | |

# Configuration

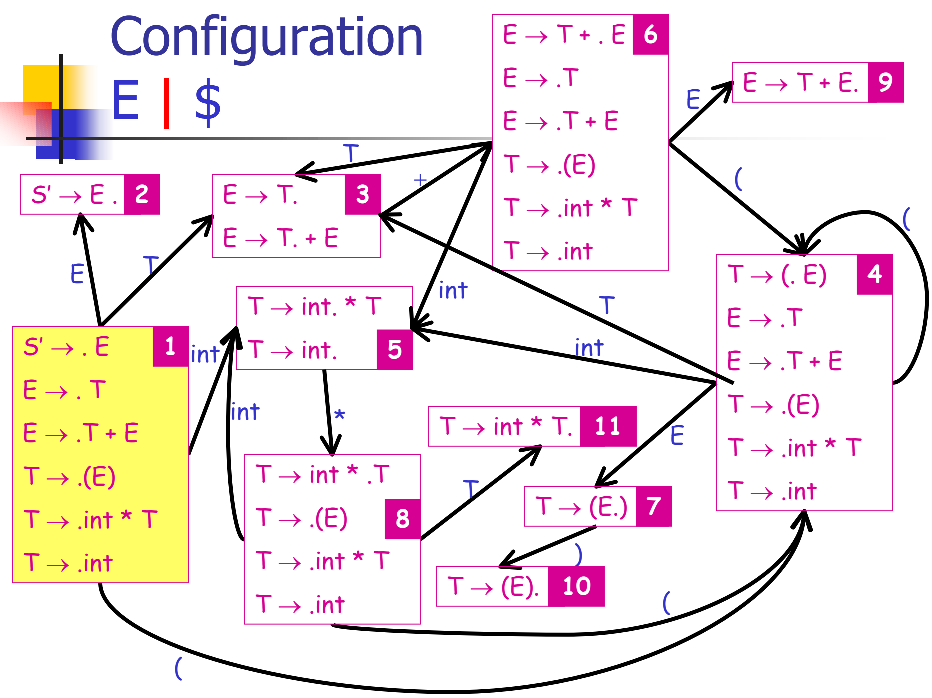
## | E $



# SLR Example

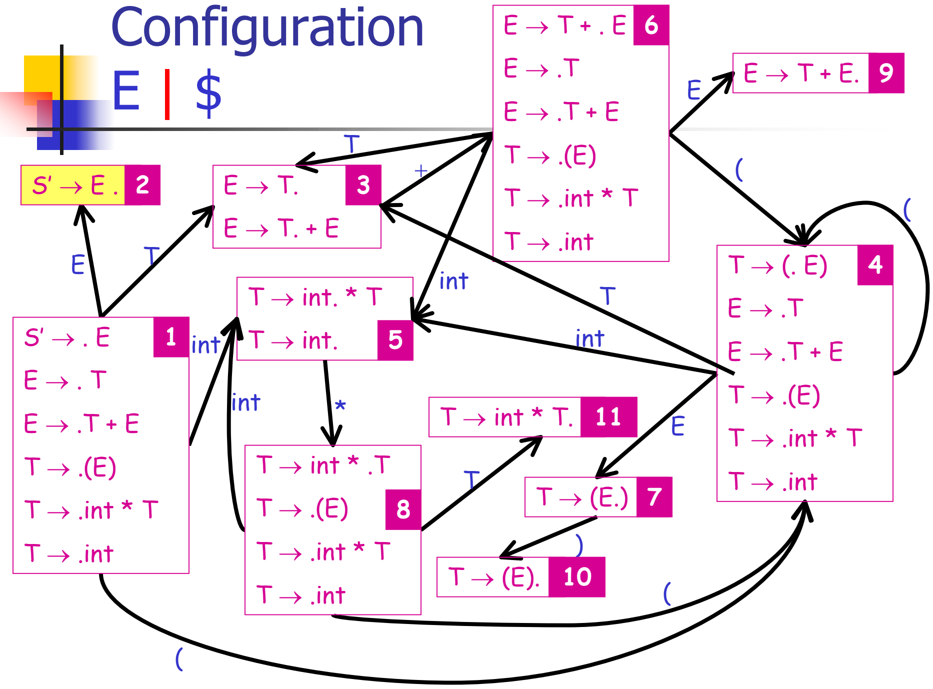
| *Configuration* | *DFA Halt State* | *Action* |
|---|---|---|
| \| int * int $ | 1 | shift |
| int \| * int $ | 5 * not in Follow(T) | shift |
| int * \| int $ | 8 | shift |
| int * int \| $ | 5 $ ∈ Follow(T) | reduce T→int |
| int * \| T $ | 8 | shift |
| int * T \| $ | 11 $ ∈ Follow(T) | reduce T→int * T |
| \| T $ | 1 | shift |
| T \| $ | 3 $ ∈ Follow(E) | reduce E→T |
| \| E $ | 1 | shift |
| E \| $ | 2 | ACCEPT |

# Configuration
# E | $



## Configuration

## E | $




## Notes

- Can also use one more state:
  - it accepts in state "S′ → E $ . "
  - i.e., it accepts in configuration E$|, not in E|$.
- Rerunning the automaton at each step is wasteful
  - Most of the work is repeated

## An Improvement

- Remember the state of the automaton on each prefix of the stack

- Change stack to contain pairs

  ⟨ DFA State , Symbol ⟩

## An Improvement (Cont.)

- For a stack

  ⟨ $state_1$, $sym_1$ ⟩ . . . ⟨ $state_n$ , $sym_n$ ⟩

  $state_n$ is the final state of the DFA on $sym_1$ … $sym_n$

- Detail: bottom of stack is ⟨start,any⟩ where
  - any is any dummy state
  - start is the start state of the DFA

## Goto Table

- Define Goto[i,A] = j if $state_i \rightarrow^A state_j$

- Goto is just the transition function of the DFA
  - One of two parsing tables

## Refined Parser Moves

- Shift x
  - Push $\langle a, x \rangle$ on the stack
  - a is current input
  - x is a DFA state
- Reduce $X \rightarrow \alpha$
  - As before
- Accept
- Error

# Action Table

For each state $s_i$ and terminal $a$

- If $s_i$ has item $X \to \alpha.a\beta$ and Goto[i,a] = j then Action[i,a] = shift j
- If $s_i$ has item $X \to \alpha.$ and $a \in$ Follow(X) and $X \neq S'$ then Action[i,a] = reduce $X \to \alpha$
- If $s_i$ has item $S' \to S.$ then action[i,$] = accept
- Otherwise, action[i,a] = error

# SLR Parsing Algorithm

```
Let Input = w$ be initial input
Let J = 1
Let DFA state 1 have item S' → .S
Let stack = ⟨ 1 , dummy ⟩
   repeat
        case action[top_state(stack),Input_J] of
                shift k:  push ⟨ k, Input_J ⟩, J++
                reduce X → A:
                    pop |A| pairs,
                    replace Input_{J-|A|} to Input_{J-1} with X
                    J = J - |A|
                accept: halt normally
                error: halt and report error
```

## Notes on SLR Parsing Algorithm

- Note that the algorithm uses only the DFA states and the input
  - The stack symbols are never used!
- However, we still need the symbols for semantic actions

## Constructing SLR states

- LR(0) state machine
  - encodes all strings that are valid on the stack
  - each valid string is a configuration, and hence corresponds to a state of the LR(0) state machine
  - each state tells us what to do (shift or reduce?)

# Example SLR Parse Table

| | int | * | + | ( | ) | $ | E | T |
|---|---|---|---|---|---|---|---|---|
| 1 | s5 | | | s4 | | | s2 | s3 |
| 2 | | | | | | acc | | |
| 3 | | | s6 | | r2 | r2 | | |
| 4 | s5 | | | s4 | | | s7 | s3 |
| 5 | | s8 | r4 | | r4 | r4 | | |
| 6 | s5 | | | s4 | | | s9 | s3 |
| 7 | | | | | s10 | | | |
| 8 | s5 | | | s4 | | | | s11 |
| 9 | | | | | r1 | r1 | | |
| 10 | | | r5 | | r5 | r5 | | |
| 11 | | | r3 | | r3 | r3 | | |

1: E $\rightarrow$ T + E
2: E $\rightarrow$ T
3: T $\rightarrow$ int * T
4: T $\rightarrow$ int
5: T $\rightarrow$(E)

# Example SLR Parse

| Stack | Input | J | Act |
|---|---|---|---|
| <1,?> | int * int $ | 1 | s5 |
| <5,int><1,?> | | 2 | s8 |
| <8,*><5,int><1,?> | | 3 | s5 |
| <5,int><8,*><5,int><1,?> | | 4 | r4 |
| <8,*><5,int><1,?> | int * T $ | 3 | s11 |
| <11,T> <8,*><5,int><1,?> | | 4 | r3 |
| <1,?> | T $ | 1 | s3 |
| <3,T><1,?> | | 2 | r2 |
| <1,?> | E $ | 1 | s2 |
| <2,E><1,?> | | 2 | acc |

# Another Example

int * (int + int) * int $