PHYS 2021 – Final Exam

Friday, May 17, 2013

Time: 120 min

Name: ____________________________________________

Instructions

- This is a closed book test. A letter-size formulae sheet and scratch paper are allowed. A calculator is allowed.

Good luck!

Physical constants:

Speed of light: \( c = 3.0 \times 10^8 \) m/s

Stephan-Boltzmann constant: \( \sigma = 5.6703 \times 10^8 \) W/m\(^2\)K\(^4\)

Boltzmann constant: \( k_B = 1.38065 \times 10^{-23} \) J/K

Wien’s displacement law constant: \( b = 2.898 \times 10^{-3} \) m·K

Planck’s constant: \( h = 6.626 \times 10^{-34} \) J·s = 4.135667 \times 10^{-15} eV·s

\[
h = 1.054572 \times 10^{-34} \text{ J·s} = 6.582 \times 10^{-16} \text{ eV·s}
\]

Mass of electron: \( m_e = 9.109 \times 10^{-31} \) kg = 510.0 keV / \( c^2 \)

Mass of proton: \( m_p = 1.67 \times 10^{-27} \) kg = 938.3 MeV / \( c^2 \)

Unified mass unit: \( u = 1.66 \times 10^{-27} \) kg = 931.5 MeV / \( c^2 \)

Electron charge: \( e = 1.602 \times 10^{-19} \) C

Coulomb force constant: \( k_e = 8.988 \times 10^9 \) N·m\(^2\)/C\(^2\)

1 eV = 1.60 \times 10^{-19} \) J

Ground state of hydrogen atom: \( E_1 = mke2e42h2=13.6 \) eV

Ground state of infinite square well: \( E_1 = \hbar 2 \pi 22mL2 \)

Integrals:

\[
-\infty e^{-x^2/a^2}dx = \pi a
\]

\[
x^2e^xdx=(x^2b^2-2xb^2+2b^3)e^x
\]

\[
x^3e^xdx=(x^3b^2-3x^2b^2+6xb^3-6b^4)e^x
\]
1.  
   (6 points) You are an observer at an outpost at the far end of the galaxy. You see a spaceship go by and measure its length to be 85 meters. Your catalogue lists the ship as being 100 m long.
   
   a.  (2 points) What is the speed of the ship?

   b.  (2 points) You continue observing the ship and see that it takes 1 year to reach its destination. How much time passes on the ship captain’s clock?

   c.  (2 points) Indicate the two measured time periods (yours and captain’s) on a space-time diagram.
2. (6 points) In a laboratory frame of reference $S$, an electron of rest mass $m_0=0.5$ MeV travelling with speed $v=0.6c$ meets a stationary positron (also $m_0=0.5$ MeV) and they briefly form an electron-positron pair.
   
   a. (2 points) What is the momentum of the pair in $S$?

   b. (2 points) The particles then annihilate, producing two photons. How much energy is released as electromagnetic waves?

   c. (2 points) Would you expect the produced photons to have the same frequencies? If not, which one would have a higher frequency? Explain.
3. (8 points) For a hydrogen atom:
   a. (2 points) Find the wavelength of the photon emitted in a transition from n=2 to n=1.

   b. (2 points) For a quantum number n=2, enumerate the distinct states that the electron can be in. What is the total number of such states?

   c. (1 point) If you measure the z-component of the electron’s orbital angular momentum, L_z, in a state with (n=4, l=3), what values can you find?

   d. (3 points) Using classical physics and a Bohr model, write the expressions for the electron’s kinetic energy and potential energy. Using these expressions, derive the expression for the radius of the first Bohr orbit.
4. (12 points) An electron is in the n=3 state with energy $E$ inside a potential well of width $L$ shown below.

a. (2 points) Sketch a possible wavefunction of this electron.

b. (2 points) What is the uncertainty in its velocity?

c. (2 points) What is the expectation value of its momentum?

d. (2 points) Estimate the probability of the electron being in a small region of space $0.495\ L < x < 0.505\ L$. 

e. (2 points) The electron is excited from the shown state \((E)\) by a photon of momentum \(p = 6 \text{ eV/c.}\) Sketch the new wavefunction of the electron.

f. (2 points) What is the speed of this excited electron at \(x=L/2\)?
5. (8 points) The electron in the ground state \((n=1, l=0)\) of hydrogen atom is characterized by the normalized radial function

\[
R_{10} = 2a_0^3 e^{-r/a_0},
\]

where \(a_0\) is the Bohr radius, and \(r\) is the distance from nucleus.

a) (4 points) Find the distance \(r\) where the electron is most likely to be found (i.e. the most probable value of \(r\)).

b) (4 points) Find the expectation value of \(r\) (i.e. the average distance of the electron from the nucleus).
6. *(3 points)* Name one atom for which the Stern-Gerlach experiment (where an atomic beam is passed through an inhomogeneous magnetic field and detected on a collector plate) would produce a single line, and one atom for which the experiment would produce two lines. Explain. Consider all atoms in ground state.