**Physical constants:**

- Speed of light: $3.0 \times 10^8$ m/s
- Stephan-Boltzmann constant: $\sigma = 5.6703 \times 10^{-8}$ W/m$^2$K$^4$
- Wien’s displacement law constant: $b = 2.898 \times 10^3$ m·K
- Planck’s constant: $h = 6.626 \times 10^{-34}$ J·s
  
  $$h = 1.054572 \times 10^{-34} \text{ J·s} = 6.582 \times 10^{-16} \text{ eV·s}$$
- Mass of electron: $m_e = 9.109 \times 10^{-31}$ kg = $510.0$ keV / $c^2$
- Mass of proton: $m_p = 1.67 \times 10^{-27}$ kg = $938.3$ MeV / $c^2$
- Unified mass unit: $u = 1.66 \times 10^{-27}$ kg = $931.5$ MeV / $c^2$
- Electron charge: $e = 1.602 \times 10^{-19}$ C
- Coulomb force constant: $k_e = 8.988 \times 10^9 \text{ N·m}^2/\text{C}^2$

$1 \text{ eV} = 1.60 \times 10^{-19}$ J

- Ground state of hydrogen atom: $E_1 = \frac{k_e^2 e^4}{2m_h^2} \approx 13.6$ eV
- Ground state of infinite square well: $E_1 = \frac{\hbar^2 \pi^2}{2ml^2}$
- Probability integral: $\int_{-\infty}^{\infty} e^{-x^2/a^2} \, dx = \sqrt{\pi} \, a$

1. A 40-W incandescent bulb has the surface area of its tungsten filament of 0.5 cm$^2$ and radiates like a blackbody.
   a. What is the wavelength at the maximum of the spectral distribution?
   b. What is the temperature of the filament?
      
      (1490 nm; 1940 K)

2. Under optimum conditions, the eye will perceive a flash if about 60 photons arrive at the cornea. How much energy is this in joules if the wavelength of the light is 550 nm?
   
   $(135 \text{ eV} = 2.17 \times 10^{-17} \text{ J})$

3. A stationary hydrogen atom initially in the first excited state ($n=2$) emits a Lyman $\alpha$ photon. Find the velocity of the atom after the emission.
   
   (3 m/s)

4. Find the velocity of photoelectrons liberated by light of wavelength $\lambda = 18.0$ nm from stationary He$^+$ ions in the ground state. You may neglect the recoil of the ion and use the non-relativistic approximation.
   
   $(2.3 \times 10^6 \text{ m/s})$
5. A dust particle of mass $m = 10^{-6}$ kg is moving with velocity $v = 10^{-6}$ m/s. If the uncertainty in the position of its wave packet is equal to its de Broglie wavelength, what is the uncertainty in its velocity? 

$(8 \times 10^{-8}$ m/s)

6. In a region of space, a particle has a wave function given by $\psi(x) = A \exp[-x^2/(2L^2)]$ and energy $E = \hbar^2/(2mL^2)$, where $L$ is some length.

   a. Find the potential energy $V(x)$ as a function of $x$.
   b. The normalization constant is $A = 0.751/\sqrt{L}$. Estimate the probability of the particle being in a small region of space $-0.005L < x < 0.005L$.
   c. Derive the above expression for the normalization constant $A$.

$(V(x)=hbar^2x^2/(2mL^2); P=5.64x10^{-3})$

7. An atom of mass $m=3 \times 10^{-26}$ kg oscillates harmonically in one dimension at an angular frequency of $\omega = 10^{13}$ rad/s.

   a. What is its ground state energy?
   b. What is the effective force constant ($k$)?

$(E_0 = 0.0033$ eV; $k=3$ N/m)

8. Sketch a possible wave function for a particle of energy $E$ trapped inside the potential well shown below.
9. An electron moving in a nanowire can be approximated as moving in a one-dimensional infinite potential well. The nanowire is $L=2$ $\mu$m long. The nanowire is cooled to a temperature of $T=13$ K, and the electron's average kinetic energy is approximately equal to that of gas molecules at this temperature: $E_k = \frac{3}{2} k_B T$, where $k_B$ is the Boltzmann constant. What is the approximate quantum number for the electrons moving in the wire? (n=134)

10. Extra credit A particle is trapped inside an infinite square-well potential between $x=0$ and $x=L$. Its wave function is a superposition of the ground state and first excited state:

$$\psi(x) = a\psi_1(x) + b\psi_2(x),$$

where $\psi_1$ and $\psi_2$ are the respective eigenstate wave functions. The wave function $\psi(x)$ is normalized. The coefficient $a$ is $a=1/2$. What is the probability of finding the particle in the first excited state? (P=3/4)