Physical constants:

Speed of light: 3.0×10^8 m/s
Stephan-Boltzmann constant: $\sigma = 5.6703\times10^{-8}$ W/m^{2}K^4
Wien's displacement law constant: $b = 2.898\times10^{-3}$ m·K
Planck's constant: $h = 6.626 \times 10^{-34}$ J·s
$\hbar = 1.054572 \times 10^{-34}$ J·s $= 6.582 \times 10^{-16}$ eV·s
Mass of electron: $m_e = 9.109 \times 10^{-31}$ kg $= 510.0$ keV / c^2
Mass of proton: $m_p = 1.67 \times 10^{-27}$ kg $= 938.3$ MeV / c^2
Unified mass unit: $u = 1.66 \times 10^{-27}$ kg $= 931.5$ MeV / c^2
Electron charge: $e = 1.602\times10^{-19}$ C
Coulomb force constant: $k_e = 8.988\times10^9$ N·m^2/C^2

1 eV = 1.60×10^{-19} J

Ground state of hydrogen atom: $E_1 = \frac{k_e^2 e^4}{2m\hbar^2} \approx 13.6$ eV
Ground state of infinite square well: $E_1 = \frac{\hbar^2\pi^2}{2mL^2}$
Probability integral: $\int_{-\infty}^{\infty} e^{-x^2/a^2} dx = \sqrt{\pi}\, a$

1. A 40-W incandescent bulb has the surface area of its tungsten filament of 0.5 cm^2 and radiates like a blackbody.
   a. What is the wavelength at the maximum of the spectral distribution?
   b. What is the temperature of the filament?

   (1490nm; 1940 K)

2. Under optimum conditions, the eye will perceive a flash if about 60 photons arrive at the cornea. How much energy is this in joules if the wavelength of the light is 550 nm?
   (135 eV = 2.17×10^{-17} J)

3. A stationary hydrogen atom initially in the first excited state (n=2) emits a Lyman α photon. Find the velocity of the atom after the emission.
   (3 m/s)

4. Find the velocity of photoelectrons liberated by light of wavelength $\lambda = 18.0$ nm from stationary He^+ ions in the ground state. You may neglect the recoil of the ion and use the non-relativistic approximation.
   (2.3 10^6 m/s)

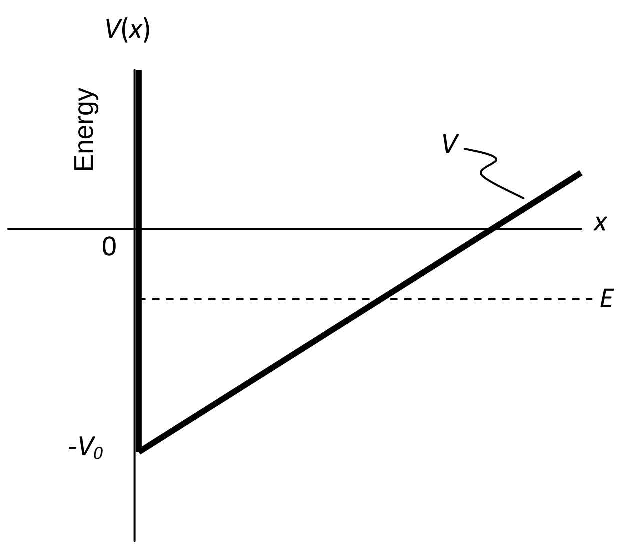
5. A dust particle of mass m= 10^{-6} kg is moving with velocity v = 10^{-6} m/s. If the uncertainty in the position of its wave packet is equal to its de Broglie wavelength, what is the uncertainty in its velocity?
   ($8x10^{-8}$ m/s)

6. In a region of space, a particle has a wave function given by $\psi(x) = A \exp[-x^2/(2L^2)]$ and energy $E=\hbar^2/2mL^2$, where L is some length.
   a. Find the potential energy $V(x)$ as a function of x.
   b. The normalization constant is $A = 0.751/\sqrt{L}$. Estimate the probability of the particle being in a small region of space $-0.005L < x < 0.005L$.
   c. Derive the above expression for the normalization constant A.
   ($V(x)=hbar^2x^2/(2mL^4)$; $P=5.64x10^{-3}$)

7. An atom of mass $m=3x10^{-26}$ kg oscillates harmonically in one dimension at an angular frequency of $\omega = 10^{13}$ rad/s.
   a. What is its ground state energy?
   b. What is the effective force constant (k)?

   (E_0 = 0.0033 eV; k=3 N/m)

8. Sketch a possible wave function for a particle of energy E trapped inside the potential well shown below.



9. An electron moving in a nanowire can be approximated as moving in a one-dimensional infinite potential well. The nanowire is L=2 μm long. The nanowire is cooled to a temperature of T=13 K, and the electron's average kinetic energy is approximately equal to that of gas molecules at this temperature: E_k =3/2 $k_B T$, where k_B is the Boltzmann constant. What is the approximate quantum number for the electrons moving in the wire?
   (n=134)

10. *Extra credit* A particle is trapped inside an infinite square-well potential between x=0 and x=L. Its wave function is a superposition of the ground state and first excited state:

    $$\psi(x) = a\ \psi_1(x) + b\ \psi_2(x),$$

    where ψ_1 and ψ_2 are the respective eigenstate wave functions. The wave function $\psi(x)$ is normalized. The coefficient a is a=1/2. What is the probability of finding the particle in the first excited state?
    (P=3/4)