PHYS 2022 - Homework 4
Due Wednesday, February 17, 2010.

Reading:
French: pp. 77-101.
Young&Freedman: 13.8 (forced oscillations and resonance).

Problems:

1. French 1-2. Hint: $(c+jd)(c-jd) = c^2+d^2 = |z_2|^2$. You can also use your previous result from problem 1-1.

2. Look at the expression for the amplitude of a forced harmonic oscillator with damping (e. g., Eq. 4-11 in French). If one sets the driving force F_0 to zero, the amplitude A also becomes zero. Why is that? Shouldn't one instead recover the expression for the amplitude of a free (undriven) oscillator?

3. Y&F 13.58. A 50.0 g hard-boiled egg moves on the end of a spring with force constant k=25.0 N/m. Its initial displacement is 0.300 m. A damping force $F_x=-bv_x$ acts on the egg, and the amplitude of the motion decreases to 0.100 m in 5.00 s. Calculate the magnitude of the damping constant b.

4. French 4-13

5. An object is making weakly damped oscillations (γ is not zero but much smaller than ω_0). On oscilloscope screen, the horizontal signal is proportional to the *displacement* of the object, and the vertical signal is proportional to the *velocity*. Draw a figure that should appear on the screen.

6. For this problem, you can use any software you like. In your calculations, write symbolic expressions first and plug in numbers in the end. You can directly use expressions that we obtained in class. An oscillator has a mass m=1 kg and a natural frequency ω_0=5 rad/s. It is driven by a force $F_0\cos(\omega_F t)$, where F_0=1 N and the frequency ω_F=4.5 rad/s. The damping factor is γ=0.2 rad/s.

a) For *steady state* oscillations, calculate the amplitude A. Calculate the phase difference δ between the oscillations and the forcing $F_0\cos(\omega_F t)$. Pay close attention to which frequencies appear in the formulas and use the correct ones. Note that, even though the driving frequency is fairly close to the resonance frequency, δ is not close to $\pi/2$. **b)** Calculate the frequency $\omega_{free\text{-}damped}$ of free (undriven) damped oscillations. Compare it to ω_0. Calculate the quality factor $Q=\omega_0/\gamma$. **c)** The oscillator makes free damped oscillations. At t=0, the amplitude is X_0=0.4m and the displacement x(t=0) is equal to X_0. At t=0, the forcing is turned on. Write the expression (using symbols rather than numbers) for the transient solution x(t). Plot this solution from t=0 to t=20*pi seconds.