PHYSICS OLYMPIAD 2014

University of Minnesota Duluth

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THEORETICAL PART

Problem 1

A chunk of Swiss cheese 10x10x10 cm has mass 650 g. When you cut a small piece off this chunk, the density of the small piece turns out to be ρ=1100 kg/m^3. This is because the big chunk has holes inside it, which are filled with gas and are not visible on the outside. What is the mass of the gas inside the cheese if the density of the gas is known to be ρ_g = 1.29 kg/m^3?

Solution: The problem does not say how small a piece was cut off, so we can assume that it is small enough to contain no gas. The density of the small piece is then the same as the density of pure cheese itself. As most of the mass in the chunk comes from cheese itself (the air inside may take substantial volume but its mass is small compared to the mass of the cheese), the volume of cheese in the chunk can be calculated as

$$V_{cheese} \approx m/\rho = 0.650/1100 = 0.5909 \times 10^{-3} \text{ m}^3.$$

Then the volume of gas inside the chunk is

$$V_g = V - V_{cheese} = 1\text{x}10^{-3} - 0.5909 \times 10^{-3} = 0.409 \text{x}10^{-3} \text{ m}^3.$$

The mass of the gas is then

$$M_g = V_g\rho_g = 0.409 \times 10^{-3} \times 1.29 = 0.528 \times 10^{-3} \text{ kg} = \underline{\mathbf{0.528\ g}}.$$

Alternatively, one can set up an equation that does not require neglecting the mass of gas in the first step above. As the mass of the chunk equals the mass of cheese plus the mass of gas, using m = ρV separately for gas and cheese and noting that the volume of cheese is the total volume minus the volume of gas, one obtains:

$$\rho_g V_g + \rho(V-V_g) = M.$$

Here, M is the total mass (650 g) and V is the total volume (1.0x10^{-3} m^3). Solving this equation for V_g, one obtains

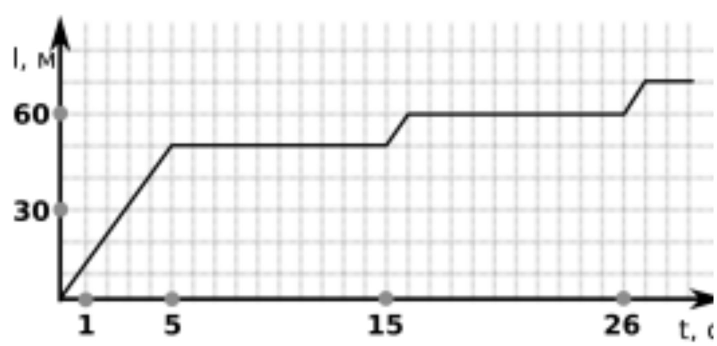
$$V_g = (\rho V-M)/(\rho-\rho_g) = 0.4096 \text{x}10^{-3} \text{ m}^3.$$

The difference from the result above is only in the last digit (<0.1%), which shows that neglecting the mass of air was a good approximation. As the numbers in the problems are stated with the accuracy of three significant digits, we only kept three significant digits in the answer, and the mass of air turns out to be the same as above, 0.528 g.

Problem 2

A road-washing vehicle is moving along a circular track that is L=400 m long. The water sprayed on the road dries up in τ = 5 sec. After each quarter-circle, the water flow is increased so that the drying up time τ increases by one second. Plot the length of the wet portion of the road as a function of time. The speed of the vehicle is V = 10 m/s.

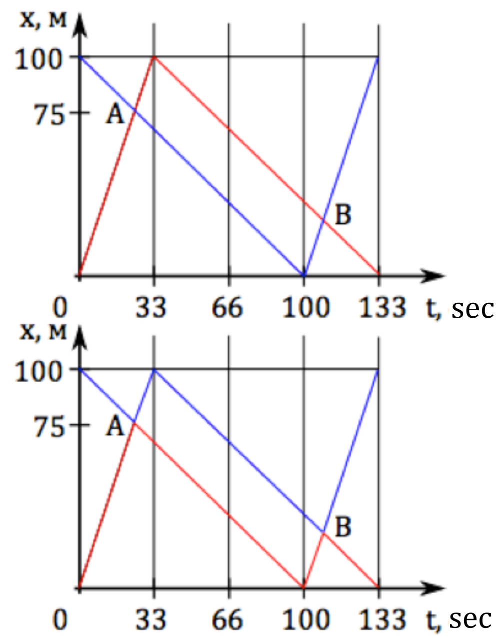
Solution: For the first 5 seconds, the vehicle travels 50 m. During this time, the length of the wet part increases at the speed of the vehicle. Then the "tail" part of it starts moving so that the length of the wet part stays equal to the distance that the vehicle passes in 5 s (50 m). After the first quarter-circle (100 m, corresponding to t=10 s), the flow of water will increase, but the length of the wet part will start increasing only as the "tail" of it reaches the point of increased water flow, which happens 5 seconds later, at 15 s. The length of the wet part will then correspond to the distance that the vehicle passes in 6 seconds (60 m). It will increase again (to 70 m) when the "tail" reaches the next quarter-circle mark, which will happen 10+1 =11 seconds later (at t=26 s). And so it will continue: each 10 seconds the length of the wet part will increase by 10 m, with the increase taking 1 second. The only other detail: after some time the entire circle will stay wet, which will happen at=365 seconds.



Problem 3

Alice and Bob are playing at an airport on a moving walkway, which moves at the speed of 1 m/s. They start at the either end of the walkway and at the same moment of time start running towards each other. Their speeds relative to the walkway are always 2 m/s. When they meet, they each turn around and run to the end of the walkway again, then immediately run towards each other again, and keep doing this over and over. What distances relative to the ground will Alice and Bob each travel in 800 sec? The length of the moving walkway is l=100 m.

Solution: Let's make a graph of the Alice and Bob's positions as functions of time. The person who runs in the direction of the moving walkway runs at 3 m/s relative to the ground, whereas the person running in the opposite direction runs at 1 m/s relative to the ground. The slopes of the graphs then should differ by a factor of 3. It's easier to first consider a slightly modified problem: let Alice and Bob run to the ends of the walkway, without turning around when they meet each other. In this case one of them reaches the end of the walkway in 33.33 seconds, whereas the other one takes 100 s. As the top graph shows, it takes each of them 133.33 seconds to return to their initial positions. Their respective distances that they traveled relative to the ground at that moment turn out to be the same.



Now let's turn to the original problem where Alice and Bob turn around when they meet. The only different that this makes is on who runs in the opposite direction, but it does not affect the velocities (see the bottom graph). The distances travelled are still the same for both of them. As in 133 seconds each of them travels 200 m, in 800 seconds each of them will travel **1,200 meters**.

EXPERIMENTAL PART

Task 1. The strength of string

Task: Measure the force that is needed to exceed the breaking strength of a string.

Equipment: The string, a small weight of known mass, graph paper, lab stand.

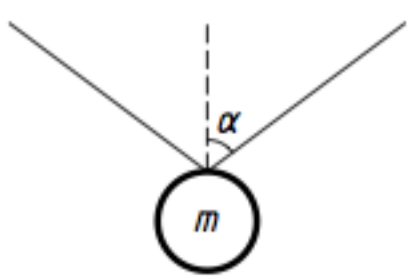
Pencils or pens can be used only for writing.

The principal difficulty in this problem is that the given weight is insufficient to break the string. Dropping the weight allows to calculate the energy at which the string breaks but it does not allow to calculate the force because it requires knowing the distance over which the energy is dissipated but the string is not stretchable, so measuring the very short distance over which the force of tension does work is not feasible. Spinning the weight or making a pendulum is not practical either, as calculating the centripetal acceleration at which the string breaks requires measuring time, and no equipment of that is given.

A practical way of measuring the breaking force is based on the idea of the decomposition of forces into components. If the weight is hung on the string and the ends of the string are pulled apart, the vertical projections of the two tension forces (in the left and right parts of the string) must balance the weight. At the point when the string breaks, the tension T can be calculated from this balance at the critical angle α:

$$2T \cos \alpha = mg.$$

The angle α can be measured by making marks on the graph paper positioned behind the string, noting the position of the string when it breaks, and doing a simple trigonometric calculation.



Task 2. The density of modeling clay

<u>Task:</u> *Find the density of a piece of modeling clay.*

<u>Equipment:</u> *Modeling clay, transparent beaker, a small jar, graph paper, thread, water.*

Measuring the density here can involve measuring separately the volume and the mass of a piece of clay. Measuring the volume is easy: for example, one can shape the clay into a cube and measure the side of the cube with the graph paper. Measuring the mass relies on the Archimedes's principle: a floating object displaces the amount of water that is equivalent in mass to its own mass. The challenge is then to make the piece of clay float and measure the amount of water that it displaces. One option is to make a "boat" out of clay and adjust its shape so that it barely floats, with its edges just above the water. Then compare the levels of water in the beaker before and after the boat is immersed in it. The levels can be measured with graph paper. Another option is to float the small jar in the beaker (attaching a piece of clay to its bottom for stability), and put the clay into the jar, noting the change in water level before and after. To improve the accuracy of the water level measurements, it helps to make the beaker effectively narrower, e.g. by filling part of its volume with clay, fingers, etc. – this will increase the height difference between water levels. Another option is to float the small jar that is partially filled with water in the beaker. Then suspend a small piece of clay on a thread and immerse it in the water inside the jar but without it touching the bottom. Then lower the clay until it rests on the bottom and mark the difference on the wall of the small jar, which will sink deeper.

The volume of the displaced water then can be calculated as the difference in height levels times the cross section (the diameter can be measured with graph paper). The mass of the water is calculated as density of water (1000 kg/m^3) times the volume. Finally, the density of clay is calculated as its mass (equal to the mass of displaced water) over its measured volume.