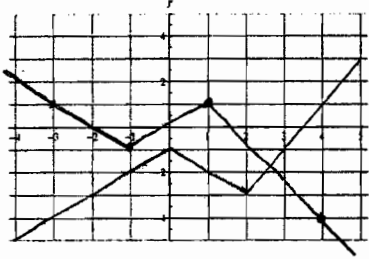


# Math 1005 exam 3

Please answer all questions carefully and show all of your work!

1. (6pts) Use the graph of  $f(x)$  to sketch  $g(x) = -f(x+1) - 2$ . Describe how the function is being changed (move left, right, up, down, stretch, shrink, flip...).



↑ flip over x-axis  
↑ left 1  
↑ down 2

2. (8pts) Let  $f(x) = \frac{1}{x-1}$  and  $g(x) = x+3$ . Combine the functions as indicated and find the domain.

a)  $(fg)(x) = \left(\frac{1}{x-1}\right)(x+3) = \frac{x+3}{x-1}$

$D_f$ : exclude when  $x-1=0 \Rightarrow x=1$   
 $(-\infty, 1) \cup (1, \infty)$

$D_g = (-\infty, \infty)$

$D_{fg} = D_f \cap D_g = (-\infty, 1) \cup (1, \infty)$

b)  $(f \circ g)(x) = f(x+3) = \frac{1}{x+3-1} = \frac{1}{x+2}$

① domain  $g: (-\infty, \infty)$

$D_{f \circ g} = ① \cap ② = (-\infty, -2) \cup (-2, \infty)$

② domain  $\frac{1}{x+2}$  exclude when  $x+2=0$   
 $x=-2$

3. (10pts) Given  $f(x) = 2x - 1$

a) Find  $f^{-1}(x)$

$y = 2x - 1$   
 $x = \frac{y+1}{2}$   
 $x+1 = 2y$   
 $\frac{x}{2} + \frac{1}{2} = y$

$f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

b) Verify that  $f(x)$  and  $f^{-1}(x)$  are inverses.

$f(f^{-1}(x)) = f\left(\frac{1}{2}x + \frac{1}{2}\right)$   
 $= 2\left(\frac{1}{2}x + \frac{1}{2}\right) - 1$   
 $= x + 1 - 1$   
 $= x \checkmark$

$f^{-1}(f(x)) = f^{-1}(2x - 1)$   
 $= \frac{1}{2}(2x - 1) + \frac{1}{2}$   
 $= x - \frac{1}{2} + \frac{1}{2}$   
 $= x \checkmark$

4. (8pts) Find the distance between the points  $(3, -1)$  and  $(-5, 0)$ .

$d = \sqrt{(-5-3)^2 + (0-(-1))^2}$   
 $= \sqrt{(-8)^2 + (1)^2} = \sqrt{64+1} = \sqrt{65} \approx 8.062$

5. (10pts) Given the equation of a circle  $x^2 + y^2 + 2x - 8 = 0$

a) Write the equation of the circle in standard form.

$$(x^2 + 2x) + y^2 = 8$$

$$(x^2 + 2x + (1)^2) + y^2 = 8 + 1$$

$$\boxed{(x+1)^2 + y^2 = 9} \Leftrightarrow (x+1)^2 + (y-0)^2 = 9$$

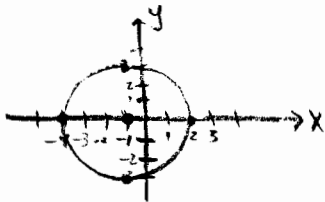
center:  $(-1, 0)$ ,  $r = \sqrt{9} = 3$

b) Identify the domain and range

$$D = [-1-3, -1+3] = \boxed{[-4, 2]}$$

$$R = [0-3, 0+3] = \boxed{[-3, 3]}$$

c) Sketch the graph



6. (8pts) Given the parabola  $f(x) = -x^2 - 2x + 3$ .

a) Does the parabola have a max or a min? Why?

$a = -1 < 0$  opens down,  $\therefore$  parabola has a max

b) Find the vertex.

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = \frac{2}{-2} = -1$$

$$\boxed{(-1, 4)}$$

$$f(-1) = -(-1)^2 - 2(-1) + 3 = -1 + 2 + 3 = 4$$

7. (8pts) Find the zeros of  $f(x) = -2x(x-1)^3(x+3)^6$ . Give the multiplicity of each zero and describe what the graph does.

$-2x = 0$	$(x-1)^3 = 0$	$(x+3)^6 = 0$
$x = 0$	$x = 1$	$x = -3$
mult 1	mult 3	mult 6
cross	cross	touch + turn

8. (6pts) Use the leading coefficient test to determine the end behavior of  $f(x) = 2x^6 + 5x^3 - 7$ .

$n = 6$  even

like  $y = x^2$

$a_n = 2 > 0$  positive

rise right and left

9. (10pts) Find all roots of  $f(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$ .

poss roots:  $\frac{\pm 1, \pm 2}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm \frac{1}{2}$

$$\begin{array}{r} \underline{1} \mid 2 \quad 3 \quad -4 \quad -3 \quad 2 \\ \quad 2 \quad 5 \quad 1 \quad -2 \\ \hline -1 \mid 2 \quad 5 \quad 1 \quad -2 \quad \underline{0} \\ \quad -2 \quad -3 \quad 2 \\ \hline \quad 2 \quad 3 \quad -2 \quad \underline{0} \end{array}$$

roots:  $x = -2, -1, \frac{1}{2}, 1$

$$2x^2 + 3x - 2 = 0$$

$$(2x-1)(x+2) = 0$$

$$2x-1=0 \quad x+2=0$$

$$x = \frac{1}{2} \quad x = -2$$

10. (10pts) Given the function  $f(x) = \frac{x+2}{x^2+3x+2} = \frac{(x+2)}{(x+2)(x+1)}$

a) Find the domain. Write in interval notation.

exclude when  $x^2+3x+2=0$   
 $(x+2)(x+1)=0$   
 $x=-2, x=-1$

$D = (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

b) Find any Vertical Asymptotes or holes

$$f(x) = \frac{(x+2)}{(x+2)(x+1)} = \frac{1}{x+1} \quad x \neq -2$$

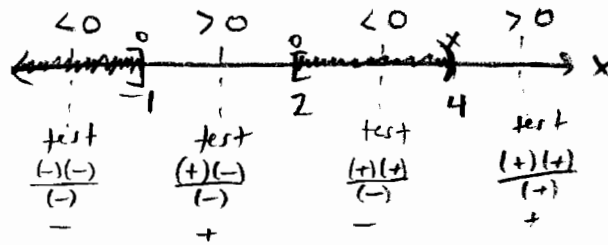
hole:  $x = -2$   
 VA:  $x = -1$

c) Find any Horizontal or Slant Asymptotes

$n=1$        $n < m$       H.A:  $y = 0$   
 $m=2$

11. (8pts) Solve  $\frac{(x+1)(x-2)}{(x-4)} \leq 0$

bdry pts  $(x+1)(x-2) = 0$   
 $x = -1, 2$   
 $x-4 = 0$   
 $x = 4$



$[-1, -1] \cup [2, 4)$

12. (8pts) Find the accumulated value of an investment of \$5000 for 3 years at an interest rate of 4% if the money is compounded semiannually.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$P = 5000, t = 3, r = 0.04, n = 2$

$$A = 5000 \left(1 + \frac{0.04}{2}\right)^{2(3)} = 5000(1.02)^6 = \$5630.81$$