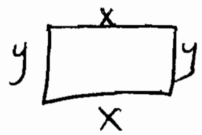
Name Key Math 1005 sec 2 Spring 2008

Participation 18 – sec 6.1-6.2

1. A rectangular lot whose perimeter is 360 feet is fenced along three sides. An expensive fencing along the lot's length costs \$20 per foot and an inexpensive fencing along the two side widths costs only \$8 per foot. The total cost of the fencing along the three sides comes to \$3280. What are the lot's dimensions?



$x =$ length

$y =$ width

$$2x + 2y = 360$$
$$20x + 8(2y) = 3280$$
$$\Rightarrow \quad 2x + 2y = 360$$
$$20x + 16y = 3280$$

$$\left[\begin{array}{cc|c} 2 & 2 & 360 \\ 20 & 16 & 3280 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 1 & 180 \\ 20 & 16 & 3280 \end{array}\right] \xrightarrow[-20R_1+R_2]{} \left[\begin{array}{cc|c} 1 & 1 & 180 \\ 0 & -4 & -320 \end{array}\right]$$

$$\xrightarrow[-\frac{1}{4}R_2]{} \left[\begin{array}{cc|c} 1 & 1 & 180 \\ 0 & 1 & 80 \end{array}\right] \xrightarrow{-R_2+R_1} \left[\begin{array}{cc|c} 1 & 0 & 100 \\ 0 & 1 & 80 \end{array}\right]$$

$x = 100$

$y = 80$

The lot is $\boxed{100 \text{ ft by } 80 \text{ ft}}$

2. A person invested \$17,000 for one year, part at 10%, part at 12%, and the remainder at 15%. The total annual income from these investments was \$2110. The amount of money invested at 12% was \$1000 less than the amount invested at 10% and 15% combined. Find the amount invested at each rate.

$x =$ amt at 10%

$y =$ amt at 12%

$z =$ amt at 15%

$$x + y + z = 17000$$
$$0.1x + 0.12y + 0.15z = 2110$$
$$x - y + z = 1000$$

(b/c $x + z = y + 1000$)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 17000 \\ 0.1 & 0.12 & 0.15 & 2110 \\ 1 & -1 & 1 & 1000 \end{array}\right] \xrightarrow[-R_1+R_2]{-0.1R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 17000 \\ 0 & 0.02 & 0.05 & 410 \\ 0 & -2 & 0 & -16000 \end{array}\right]$$

$$\xrightarrow{R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 17,000 \\ 0 & 1 & 2.5 & 20,500 \\ 0 & -2 & 0 & -16,000 \end{array}\right] \xrightarrow[2R_2+R_3]{-R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1.5 & -3500 \\ 0 & 1 & 2.5 & 20,500 \\ 0 & 0 & 5 & 25,000 \end{array}\right]$$

$$\xrightarrow[\frac{1}{5}R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & -1.5 & -3500 \\ 0 & 1 & 2.5 & 20,500 \\ 0 & 0 & 1 & 5,000 \end{array}\right] \xrightarrow[-2.5R_3+R_2]{1.5R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4000 \\ 0 & 1 & 0 & 8000 \\ 0 & 0 & 1 & 5000 \end{array}\right]$$

$x = 4000$

$y = 8000$

$z = 5000$

$\boxed{\$4000 \text{ at } 10\%,\ 8000 \text{ at } 12\%,\ \$5000 \text{ at } 15\%}$

3. A company that manufactures products A, B, and C does both manufacturing and testing. The hours needed to manufacture each product is 7, 6, and 3 respectively. The hours needed to test each product is 2, 2, and 1 respectively. The company has exactly 67 hours per week available for manufacturing and 20 hours per week available for testing. Give two different combinations for the number of products that can be manufactured and tested weekly.

$x = \#$ product A

$y = \#$ product B

$z = \#$ product C

$$7x + 6y + 3z = 67$$
$$2x + 2y + z = 20$$

$$\left[\begin{array}{ccc|c} 7 & 6 & 3 & 67 \\ 2 & 2 & 1 & 20 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & 2 & 1 & 20 \\ 7 & 6 & 3 & 67 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 10 \\ 7 & 6 & 3 & 67 \end{array}\right] \xrightarrow[-7R_1+R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 10 \\ 0 & -1 & -\frac{1}{2} & -3 \end{array}\right]$$

$$\xrightarrow[-R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 10 \\ 0 & 1 & \frac{1}{2} & 3 \end{array}\right] \xrightarrow{-R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & \frac{1}{2} & 3 \end{array}\right]$$

$x = 7$

$y + \frac{1}{2}z = 3 \rightarrow y = -\frac{1}{2}z + 3$

z arbitrary

$$\boxed{\text{soln } \left(7, -\tfrac{1}{2}z + 3, z\right)}$$

Two combinations: $z = 2$ $(7, 2, 2)$

(many possible) $z = 4$ $(7, 1, 4)$