Periodic Behavior in a Class of Second Order Recurrence Relations Over the Integers

by

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Abstract:
Suppose we start a sequence with \( a_0 = 4 \) and \( a_1 = -13 \), and define the rest of the sequence using the relation

\[
a_n = \begin{cases} 
\frac{1}{64} (2a_{n-1} - 6a_{n-2}), & \quad \frac{1}{64} (2a_{n-1} - 6a_{n-2}) \in \mathbb{Z} \\
(2a_{n-1} - 6a_{n-2}), & \quad \text{otherwise.}
\end{cases}
\]

The resulting sequence is 4, -13, -50, -22, 4, 140, 4, -13, ... When a sequence repeats after \( k \) terms, we call the solution periodic with period \( k \). Thus the sequence above is periodic with period six.

In general, given the system

\[
a_n = \begin{cases} 
x(Pa_{n-1} - Qa_{n-2}), & \quad x(Pa_{n-1} - Qa_{n-2}) \in \mathbb{Z} \\
Pa_{n-1} - Qa_{n-2}, & \quad \text{otherwise},
\end{cases}
\]

where \( x \) is a rational number, \( P \) and \( Q \) are integers, we are interested in finding when periodic solutions occur. In other words, we want \( x \) values and initial conditions for specific values of \( P \) and \( Q \) which lead to a periodic solution. Using a common linear algebra method for solving recurrence relations, we developed a search method. The method and results will be outlined in this talk.

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