m1290_Exam1_F11

Tuesday, October 04, 2011
8:49 PM

$\bar{x} = 77.6$

Math 1290 – Fall 2011 – Exam 1 Name: Key

1. The number of mosquitoes (M) that end up in a room is a function of how much the window is open (W, in square feet) according to $M(W) = 5W + 2$. The number of bites (B) depends on the number of mosquitoes according to $B(M) = 0.5M$. Find the number of bites as a function of how much the window is open. How many bites would you get if the window were 10 cm^2 open?

$B(M(W)) = B(5W+2) = 0.5(5W+2) = 2.5W+1$ J+4

$B(M(10)) = 2.5(10)+1 = \boxed{26 \text{ bites}}$ J+6

2. We have added a new garden and need to buy mulch. The garden measures 134 ft^2 and we wish to cover it with 4 inches of mulch. However, mulch is sold by the cubic yard and we can buy it in half yard increments. How many cubic yards (to the nearest half yard) do we need? (1 yard = 3 feet)

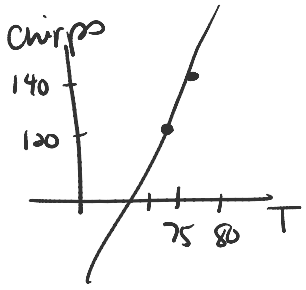
4 in $= \frac{1}{3}$ ft $= 0.333$ ft

Volume need $= (0.333)(134) = 44.667 \text{ ft}^3 \left| \frac{1 \text{ yd}^3}{(3\text{ft})^3} \right. = 1.65 \text{ yd}^3$

+4

$\boxed{\text{So, buy } 2 \text{ yd}^3}$

3. One evening you hear a cricket chirping 140 times in a minute and notice the temperature is 80°F. Later in the evening, the cricket has slowed down to 120 chirps per minute when the temperature has dropped to 75°F. If the number of chirps a cricket makes per minute is linearly related to the temperature, determine the temperature that induces the cricket to chirp 90 times in a minute.



$m = \frac{140-120}{80-75} = \frac{20}{5} = 4$ J+3

$y = 4x + b$
$140 = 4(80) + b$
$140 = 320 + b$
$b = -180$

$y = 4x - 180$ J+2

$90 = 4x - 180$ J+3

$270 = 4x$

$\boxed{x = 67.5°F}$ J+1

or $y = 0.25x + 45$
where x = # of chirps & y = Temp

4. Suppose that a population of bacteria starts with 4 million members and triples every hour. If 3 million are removed **after** reproduction, write the DDS that describes this population.

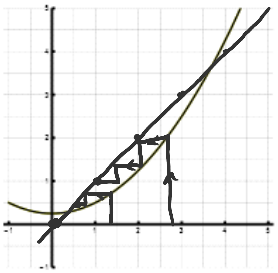
$b_{t+1} = 3b_t - 3\times10^6$

(where x = # of chirps ; y = Temp)

4. Suppose that a population of bacteria starts with 4 million members and triples every hour. If 3 million are removed **after** reproduction, write the DDS that describes this population.

$$b_{t+1} = 3b_t - 3\times10^6$$

5. Estimate the equilibria of the DDS from the graph of the updating function. Use cobwebbing to determine if each equilibria is stable or unstable.



fixed points: +5

+2 $x^* \approx 0.4$ (stable) +3

+2 $x^* \approx 3.6$ (unstable)

6. A culture of bacteria has a population of 150 cells initially. The population doubles every 12 hours, which means the population size is described by $p(t) = 150 \cdot 2^{t/12}$, where t is the time in hours. How long does it take the population to triple in size?

Triples: $3 \cdot 150 = 450$

$$\frac{450}{150} = \frac{150 \cdot 2^{\frac{t}{12}}}{150}$$

$$3 = 2^{\frac{t}{12}}$$

+2 $\ln(3) = \ln(2^{\frac{t}{12}})$

+2 $\ln(3) = \frac{t}{12}\ln(2)$

$$t = \frac{\ln(3)}{\ln(2)} \cdot 12$$

$$\boxed{t \approx 19.01 \text{ hours}}$$ +1

7. Write the equation for the cosine curve that has the following properties:
Average=2, maximum =5, minimum =-1, period = 3, phase shift = 2(right)

$$y = 2 + 3\cos\left(\frac{2\pi}{3}(t - 2)\right)$$

+2 +2 +2 +2 +2

8. Find the inverse function, $f^{-1}(x)$, for the function $f(x) = 2 + e^{4x}$. Find the domain and range of $f^{-1}(x)$.

+2 +2

$y = 2 + e^{4x}$

$y - 2 = e^{4x}$

$\ln(y-2) = \ln(e^{4x})$ ✓+3

$\ln(y-2) = 4x$

$x = \frac{\ln(y-2)}{4}$

$f^{-1}(x) = \frac{\ln(x-2)}{4}$ ✓+3

$D_f = (-\infty, \infty) = R_{f^{-1}}$

$R_f = (2, \infty) = D_{f^{-1}}$

9. Recall that the **DDS** for the lung with absorption is $c_{t+1} = (1-q)(1-\alpha)c_t + q\gamma$ where c_t is the chemical concentration in the lung at time t, γ is the ambient chemical concentration, q is the fraction of air exchanged and α is the fraction of chemical absorbed. Find the equilibrium lung concentration with the following parameter values: $\alpha = 0.4$ $\gamma = 6\frac{mmol}{L}$, the volume of the lung V=2 L, and the amount breathed in and out is W=0.5 L.

$C_{t+1} = \left(1 - \frac{0.5}{2}\right)(1 - .4)C_t + \left(\frac{0.5}{2}\right)6$

$C_{t+1} = (0.75)(0.6)C_t + (0.25)6$

$C_{t+1} = 0.45C_t + 1.5$ ✓+3

Let $C^* = C_t = C_{t+1}$ ✓+2

$C^* = 0.45C^* + 1.5$ ✓+3

$0.55C^* = 1.5$

$C^* = 2.7\overline{27}$ ✓+2

10. Given $f(x) = x^2 + 4$

a) Find the average rate of change of *f(x)* with base point $x_0 = 2$ *and* $\Delta x = 0.5$.

+2

avg rate of change $= \frac{\Delta f}{\Delta x} = \frac{f(2+0.5) - f(2)}{0.5}$

$= \frac{10.25 - 8}{0.5} = 4.5$ ✓+3

b) Estimate the slope of the line tangent to the curve, $f(x) = x^2 + 4$, at $x_0 = 2$.

let Δx get smaller & smaller

$\Delta x = 0.1$ $\frac{\Delta f}{\Delta x} = \frac{f(2.1) - f(2)}{0.1} = \frac{8.41 - 8}{0.1} = 4.1$ ✓+1.5

$\Delta x = 0.01$ $\frac{\Delta f}{\Delta x} = \frac{f(2.01) - f(2)}{0.01} = \frac{8.0401 - 8}{0.01} = 4.01$ ✓+1.5

It seems these values are limiting to 4. ✓+2