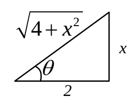
$$\int \frac{1}{\sqrt{4+x^2}}\,dx$$

1

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = \int \frac{1}{2\sec\theta} 2\sec^2\theta d\theta \qquad \begin{matrix} x = 2\tan\theta \\ dx = 2\sec^2\theta d\theta \end{matrix}$$

$$= \int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$$



1

$$\int \tan x \sec^4 x dx$$

1

$$\int \tan x \sec^4 x dx = \int \tan x \sec^2 x \sec^2 x dx =$$

$$\int \tan x (1+\tan^2 x)\sec^2 x dx = \int u(1+u^2)du \qquad \begin{matrix} u = \tan x \\ du = \sec^2 x dx \end{matrix}$$

$$\int (u+u^3)du = \frac{1}{2}u^2 + \frac{1}{4}u^4 + C = \frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + C$$

$-or-$

$$\int \tan x \sec^4 x dx = \int \tan x \sec x \sec^3 x dx = \int u^3 du \qquad \begin{matrix} u = \sec x \\ du = \sec x \tan x dx \end{matrix}$$

$$= \frac{1}{4}u^4 + C = \frac{1}{4}\sec^4 x + C$$

1

$$\int \cos^3 x \sin^2 x dx$$

1

$$\int \cos^3 x \sin^2 x dx = \int \cos^2 x \cos x \sin^2 x dx =$$

$$\int (1-\sin^2 x)\cos x \sin^2 x dx = \int (u^2 - u^4)du \qquad \begin{matrix} u = \sin x \\ du = \cos x dx \end{matrix}$$

$$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$$

1

$$\int \sqrt{1-x^2}\,dx$$

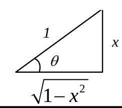
1

$$\int \sqrt{1-x^2}\,dx = \int \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta \qquad \begin{aligned} x &= \sin\theta \\ dx &= \cos\theta\, d\theta \end{aligned}$$

$$= \int \cos\theta\cos\theta\, d\theta = \int \cos^2\theta\, d\theta = \int \frac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C = \frac{1}{2}\theta + \frac{1}{4}(2\sin\theta\cos\theta) + C$$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C = \frac{1}{2}\sin^{-1}x + \frac{1}{2}x\sqrt{1-x^2} + C$$



1

$$\int \frac{x}{\sqrt{x^2-4}}\,dx$$

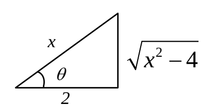
using $\tan^2\theta = \sec^2\theta - 1$

1

$$\int \frac{x}{\sqrt{x^2-4}}\,dx = \int \frac{2\sec\theta}{\sqrt{(2\sec\theta)^2-4}}\,2\sec\theta\tan\theta\, d\theta \qquad \begin{aligned} x &= 2\sec\theta \\ dx &= 2\sec\theta\tan\theta\, d\theta \end{aligned}$$

$$= 4\int \frac{\sec^2\theta}{2\tan\theta}\tan\theta\, d\theta = 2\int \sec^2\theta\, d\theta = 2\tan\theta + C = 2\left(\frac{\sqrt{x^2-4}}{2}\right) + C$$

$$= \sqrt{x^2-4} + C$$



Do you get the same answer if you solve this using a u-substitution? I hope so!

1

$$\int \frac{\cos x}{\sqrt{5+\sin^2 x}}\,dx$$

1

$$\int \frac{\cos x}{\sqrt{5+\sin^2 x}}\,dx = \int \frac{1}{\sqrt{5+u^2}}\,du \overset{a=\sqrt{5}}{\underset{\#25}{=}} \ln|u+\sqrt{5+u^2}\,| + C$$

$$= \ln|\sin x + \sqrt{5+\sin^2 x}\,| + C$$

side:
$u = \sin x$
$du = \cos x\,dx$

1

$$\int \frac{2}{x^2 - 16}\,dx$$

2

$$\int \frac{2}{x^2-16}\,dx = \int \frac{2}{(x-4)(x+4)}\,dx = \int \frac{A}{x+4} + \frac{B}{x-4}\,dx$$

$$= \int \frac{\frac{-1}{4}}{x+4} + \frac{\frac{1}{4}}{x-4}\,dx = \frac{-1}{4}\ln|x+4| + \frac{1}{4}\ln|x-4| + C$$

$$\frac{1}{4}(-\ln|x+4| + \ln|x-4|) + C = \frac{1}{4}\ln\left|\frac{x-4}{x+4}\right| + C$$

Can you generalize this to prove: $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{a}\ln\left|\frac{x-a}{x+a}\right| + C$?

2

$$\int \frac{2x+3}{x^2+2x+1}\,dx$$

2

$$\int \frac{2x+3}{x^2+2x+1}\,dx = \int \frac{2x+3}{(x+1)^2}\,dx = \int \frac{A}{x+1} + \frac{B}{(x+1)^2}\,dx$$

$$= \int \frac{2}{x+1} + \frac{1}{(x+1)^2}\,dx = 2\ln|x+1| - \frac{1}{x+1} + C$$

2

$$\int \frac{x^3}{x^2-2x+1}\,dx$$

2

by long division

$$\int \frac{x^3}{x^2-2x+1}\,dx = \int x+2+\frac{3x-2}{x^2-2x+1}\,dx = \int x+2+\frac{3x-2}{(x-1)^2}\,dx$$

$$= \int x+2+\frac{A}{x-1}+\frac{B}{(x-1)^2}\,dx = \int x+2+\frac{3}{x-1}+\frac{1}{(x-1)^2}\,dx =$$

$$= \frac{1}{2}x^2 + 2x + 3\ln|x-1| - \frac{1}{x-1} + C$$

2

$$\int \frac{4x+1}{x^3+x}dx$$

2

$$\int \frac{4x+1}{x^3+x}dx = \int \frac{4x+1}{x(x^2+1)}dx = \int \frac{A}{x}+\frac{Bx+C}{x^2+1}dx$$

$$= \int \frac{1}{x}+\frac{-1x+4}{x^2+1}dx = \int \frac{1}{x}+\frac{-1x}{x^2+1}+\frac{4}{x^2+1}dx$$

$$= \ln|x| - \frac{1}{2}\ln|x^2+1| + 4\tan^{-1}(x) + C$$

$$side: \int \frac{-1x}{x^2+1}dx = -\int \frac{1}{u}\frac{1}{2}du \quad \begin{matrix} u = x^2+1 \\ du = 2xdx \end{matrix}$$

$$-\frac{1}{2}\int \frac{1}{u}du = -\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln|x^2+1| + C$$

2

$$\int \frac{x-1}{x^2+3}dx$$

2

$$\int \frac{x-1}{x^2+3}dx = \int \frac{x}{x^2+3} - \frac{1}{x^2+3}dx = \frac{1}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$side: \int \frac{x}{x^2+3}dx = \int \frac{1}{u}\frac{1}{2}du \quad \begin{matrix} u = x^2+3 \\ du = 2xdx \end{matrix}$$

$$\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+3| + C$$

2

Use the comparison test to determine if the following converges or diverges.

$$\int_1^\infty \frac{1}{\sqrt{x}+x^3}dx$$

2

$$\int_1^\infty \frac{1}{\sqrt{x}+x^3}dx$$

$$Since \ \frac{1}{\sqrt{x}+x^3} < \frac{1}{x^3}$$

and

$$\int_1^\infty \frac{1}{x^3}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^3}dx = \lim_{t\to\infty}\int_1^t x^{-3}dx = \lim_{t\to\infty} -\frac{1}{2}x^{-2}\Big|_1^t$$

$$= \lim_{t\to\infty} -\frac{1}{2}t^{-2} + \frac{1}{2}(1)^{-2} = \frac{1}{2}$$

$$Thus, \int_1^\infty \frac{1}{\sqrt{x}+x^3}dx \text{ also converges.}$$

2

$$\int \cos^2(3x)dx$$

3

$$\int \cos^2(3x)dx = \int \frac{1}{2}(1+\cos(2(3x)))dx = \frac{1}{2}\int (1+\cos(6x))dx =$$
$$\frac{1}{2}[x+\frac{1}{6}\sin(6x)]+C = \frac{1}{2}x+\frac{1}{12}\sin(6x)+C$$

3

$$\int xe^{3x}dx$$

3

Integration by parts

$$u = x \qquad dv = e^{3x}dx$$
$$du = dx \qquad v = \frac{1}{3}e^{3x}$$

$$\int xe^{3x}dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}dx$$
$$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{3}\left(\frac{1}{3}e^{3x}\right)+C$$
$$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}+C$$

3

$$\int \frac{x}{1+\sqrt{x}}dx$$

3

$$u = \sqrt{x} \qquad u^2 = x \qquad 2udu = dx$$

$$\int \frac{x}{1+\sqrt{x}}dx = \int \frac{u^2 2udu}{1+u}$$
$$= 2\int \frac{u^3}{1+u}du = 2\int u^2 - u + 1 - \frac{1}{1+u}du \ \text{(by long division)}$$
$$= 2\left[\frac{1}{3}u^3 - \frac{1}{2}u^2 + u - \ln|u+1|\right]+C$$
$$= 2\left[\frac{1}{3}\sqrt{x}^3 - \frac{1}{2}\sqrt{x}^2 + \sqrt{x} - \ln|\sqrt{x}+1|\right]+C$$
$$= \frac{2}{3}x^{2/3} - x + 2\sqrt{x} - 2\ln|\sqrt{x}+1| + C$$

3

$$\int \frac{e^{2x}}{1+e^{x}}\,dx$$

3

$$\int \frac{e^{2x}}{1+e^{x}}\,dx \qquad \text{let } u = e^{x}, \quad \frac{du}{u} = dx$$

$$= \int \frac{u^2 \frac{du}{u}}{1+u} = \int \frac{u}{1+u}\,du = \int 1 - \frac{1}{u+1}\,du \ \ (\text{by long division})$$

$$= u - \ln|u+1| + C$$

$$= e^{x} - \ln|e^{x}+1| + C$$

3

$$\int \frac{\sqrt{4-(\ln x)^2}}{x}\,dx$$

3

$$\int \frac{\sqrt{4-(\ln x)^2}}{x}\,dx = \int \sqrt{4-u^2}\,du \qquad u = \ln x, \quad du = \frac{1}{x}\,dx$$

$$\underset{a=2}{\overset{30}{=}} \frac{u}{2}\sqrt{4-u^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{u}{2}\right) + C$$

$$= \frac{\ln x}{2}\sqrt{4-(\ln x)^2} + 2\sin^{-1}\left(\frac{\ln x}{2}\right) + C$$

or

$$\int \sqrt{4-u^2}\,du = \int \sqrt{4-(2\sin\theta)^2}\,2\cos\theta\, d\theta \qquad u = 2\sin\theta, \quad du = 2\cos\theta\, d\theta$$

$$= 4\int \cos^2\theta\, d\theta = 4\int \frac{1}{2}(1+\cos(2\theta))\,d\theta = 2\int (1+\cos(2\theta))\,d\theta$$

$$= 2[\theta + \frac{1}{2}\sin(2\theta)] + C = 2\theta + 2\sin\theta\cos\theta + C$$

$$= 2\sin^{-1}\frac{u}{2} + u\frac{\sqrt{4-u^2}}{2} + C == 2\sin^{-1}\frac{\ln x}{2} + (\ln x)\frac{\sqrt{4-(\ln x)^2}}{2} + C$$

3

$$\int_0^{10} \frac{1}{\sqrt[3]{x}}\,dx$$

3

$$\int_0^{10} \frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to 0^+}\int_t^{10} \frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to 0^+}\int_t^{10} x^{\frac{-1}{3}}\,dx = \lim_{t\to 0^+} \frac{3}{2}x^{\frac{2}{3}}\Big|_t^{10}$$

$$= \lim_{t\to 0^+} \frac{3}{2}(10)^{\frac{2}{3}} - \frac{3}{2}t^{\frac{2}{3}} = \frac{3}{2}(10)^{\frac{2}{3}} \approx 6.96 \quad (\textit{converges})$$

3

Find the arc length of the function

$$x=\frac{y^3}{6}+\frac{1}{2y}\ \ from\ \ y=2\ to\ y=3.$$

4

$$x=\frac{y^3}{6}+\frac{1}{2y}\ \ from\ \ y=2\ to\ y=3$$

$$\frac{dx}{dy}=\frac{y^2}{2}-\frac{1}{2y^2}\quad \left(\frac{dx}{dy}\right)^2=\frac{y^4}{4}-\frac{1}{2}+\frac{1}{4y^4}$$

$$\left(\frac{dx}{dy}\right)^2+1=\frac{y^4}{4}+\frac{1}{2}+\frac{1}{4y^4}=\left(\frac{y^2}{2}+\frac{1}{2y^2}\right)^2$$

$$L=\int_2^3\sqrt{\left(\frac{y^2}{2}+\frac{1}{2y^2}\right)^2}\,dy=\int_2^3\left(\frac{y^2}{2}+\frac{1}{2y^2}\right)dy=$$

$$=\frac{1}{2}\int_2^3\left(y^2+y^{-2}\right)dy=\frac{1}{2}\left[\frac{y^3}{3}-\frac{1}{y}\right]_2^3=\frac{13}{4}=3.25$$

4

Use Simpson’s Rule with n=4 to approximate the arc length of

$$y=\tan x\ \ \ from\ 0\le x\le\frac{\pi}{3}$$

accurate to 2 decimal places.

4

$$y=\tan x\ \ from\ 0\le x\le\frac{\pi}{3}$$

$$\frac{dy}{dx}=\sec^2 x\ \left(\frac{dy}{dx}\right)^2=\sec^4 x$$

$$L=\int_0^{\frac{\pi}{3}}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_0^{\frac{\pi}{3}}\sqrt{1+\sec^4 x}dx\ \ let\ f(x)=\sqrt{1+\sec^4 x}$$

$$\Delta x=\frac{b-a}{n}=\frac{\frac{\pi}{3}-0}{4}=\frac{\pi}{12}$$

$$L\approx S_4=\frac{\Delta x}{3}[1f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+1f(x_4)]$$

$$=\frac{\frac{\pi}{12}}{3}[1f(0)+4f(\frac{\pi}{12})+2f(\frac{2\pi}{12})+4f(\frac{3\pi}{12})+1f(\frac{4\pi}{12})]$$

$$=\frac{\pi}{36}[1f(0)+4f(\frac{\pi}{12})+2f(\frac{2\pi}{12})+4f(\frac{3\pi}{12})+1f(\frac{4\pi}{12})]$$

$$=\frac{\pi}{36}[1.41+5.86+13.33+8.94+4.12]\approx 2.93$$

4

Find the surface area when $y=\frac{1}{3}x^3$ on $0\le x\le 1$ is revolved about the x-axis.

4

$$y=\frac{1}{3}x^3\ \ \frac{dy}{dx}=x^2\ \left(\frac{dy}{dx}\right)^2=x^4$$

$$S=\int_0^1 2\pi(radius)(arclength)dx=2\pi\int_0^1(y)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=$$

$$=2\pi\int_0^1\frac{1}{3}x^3\sqrt{1+x^4}\,dx=\frac{2\pi}{3}\int_1^2\frac{1}{4}\sqrt{u}du\quad \begin{matrix}u=1+x^4 & u(0)=1+0^4=1\\ du=4x^3dx & u(1)=1+1^4=2\end{matrix}$$

$$\frac{2\pi}{12}\int_1^2 u^{1/2}du=\frac{2\pi}{12}[\frac{2}{3}u^{3/2}]_1^2=\frac{\pi}{9}[2^{3/2}-1^{3/2}]\approx 0.64$$

4

Find the surface area when $y=\sqrt{x+1}$ is revolved about the x axis from $1 \le x \le 5$.

4

$$y=\sqrt{x+1} \to \frac{dy}{dx}=\frac{1}{2\sqrt{x+1}} \to \left(\frac{dy}{dx}\right)^2=\frac{1}{4(x+1)}$$

$$SA=\int_1^5 2\pi\sqrt{x+1}\sqrt{1+\frac{1}{4(x+1)}}dx$$

$$=2\pi\int_1^5\sqrt{(x+1)+\frac{1}{4}}dx=2\pi\int_1^5\sqrt{x+\frac{5}{4}}dx \quad let\, u=x+\frac{5}{4}$$

$$du=dx$$

$$=2\pi\int_{9/4}^{25/4}\sqrt{u}\,du=2\pi\left[\frac{2}{3}u^{3/2}\right]_{9/4}^{25/4}=\frac{49\pi}{3}\approx 51.3$$

4

Set up the **integrals** to determine the surface area when the function $y=4-x^2$ from (-3,-5) to (-1,3) is revolved about the **y axis** by integrating as a function of

a) x

b) y

4

$$a)\ y=4-x^2 \quad \frac{dy}{dx}=-2x \quad \left(\frac{dy}{dx}\right)^2=4x^2$$

$$SA=\int_{-3}^{-1}2\pi(radius)\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx=\int_{-3}^{-1}2\pi x\sqrt{1+4x^2}dx$$

$$b)\ y=4-x^2 \quad x=\sqrt{4-y} \quad \frac{dx}{dy}=\frac{-1}{2}(4-y)^{-1/2}(-1) \quad \left(\frac{dx}{dy}\right)^2=\frac{1}{4(4-y)}$$

$$SA=\int_{-5}^{3}2\pi(radius)\sqrt{1+\left(\frac{dx}{dy}\right)^2}dy=\int_{-5}^{3}2\pi\sqrt{4-y}\sqrt{1+\frac{1}{4(4-y)}}dy$$

4

$$Evaluate: \int_0^\infty \frac{1}{(2x+5)^4}dx$$

4

$$\int_0^\infty\frac{1}{(2x+5)^4}dx=\lim_{t\to\infty}\int_0^t\frac{1}{(2x+5)^4}dx=\lim_{t\to\infty}\left.\frac{-1}{6(2x+5)^3}\right|_0^t$$

$$=\lim_{t\to\infty}\left[\frac{-1}{6(2t+5)^3}-\frac{-1}{6(2(0)+5)^3}\right]=0+\frac{1}{6(5)^3}=\frac{1}{750} \quad (converges)$$

$$side: u=2x+5$$

$$\frac{1}{2}du=dx$$

$$So, \int\frac{1}{u^4}\frac{1}{2}du$$

$$=\frac{1}{2}\int u^{-4}du=\frac{-1}{6}u^{-3}=\frac{-1}{6(2x+5)^3}$$

4

Find the x coordinate of the centroid of the region bounded by:

$$y = \frac{2}{x^2}, y = 0, x = 1 \text{ and } x = 2.$$

5

$$y = \frac{2}{x^2}, y = 0, x = 1 \text{ and } x = 2$$

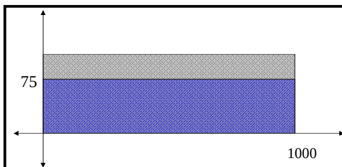
$$\bar{x} = \frac{M_y}{m} = \frac{1}{A}\int_a^b x \cdot f(x)dx \qquad A = \int_1^2 \frac{2}{x^2}dx = \frac{-2}{x}\Big|_1^2 = 1$$

$$\bar{x} = \frac{1}{1}\int_1^2 x \cdot \frac{2}{x^2}dx = \int_1^2 \frac{2}{x}dx = 2\ln|x| \,\Big|_1^2 = 2\ln 2 - 2\ln 1 = 2\ln 2$$

5

A vertical dam in the shape of a rectangle is 1000 ft wide and 100 feet high. Calculate the force on the dam when the water level is 75 feet high. Recall δ=62.5 lbs/ft^3

5



$w_{i=}$*1000 for all heights*

y_i	d_i
0	75
75	0

$$F = \int_0^{75} \delta w_i d_i dy = \int_0^{75} 62.5(1000)(75-y)dy =$$

$$= 62500\int_0^{75}(75-y)dy = 62500[75y - 0.5y^2]_0^{75} =$$

$$= 62500[2812.5 - 0] = 1.76 \times 10^8 \ lbs$$

5

Evaluate

$$\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}dx \ \text{ hint}: usub$$

5

Note: This is an improper integral because $\sqrt{0^2 + 2(0)} = 0$

$$\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}dx = \lim_{t\to 0^+}\int_t^1 \frac{x+1}{\sqrt{x^2+2x}}dx = \lim_{t\to 0^+}\sqrt{x^2+2x}\Big|_t^1 = \lim_{t\to 0^+}\left(\sqrt{1^2+2(1)} - \sqrt{t^2+2t}\right) = \sqrt{3} - \sqrt{0} = \sqrt{3}$$

$$Thus, \int_0^1 \frac{x+1}{\sqrt{x^2+2x}}dx \text{ converges to } \sqrt{3}.$$

side:

$$u = x^2 + 2x \qquad \int \frac{x+1}{\sqrt{x^2+2x}}dx = \int \frac{\frac{1}{2}du}{\sqrt{u}}$$

$$\frac{du}{dx} = 2x + 2 \qquad = \frac{1}{2}\int u^{\frac{-1}{2}}du = u^{\frac{1}{2}} + C$$

$$\frac{du}{dx} = 2(x+1)$$

$$\frac{1}{2}du = (x+1)dx$$

5

What is the maximal error incurred when Simpson's rule with n=6 is used to approximate

$$\int_5^8 -2x^5 dx?$$

5

$$\int_5^8 -2x^5 dx \approx S_6$$

$$f'(x) = -10x^4,\ f''(x) = -40x^3,\ f'''(x) = -120x^2,\ f^{(4)}(x) = -240x$$

$$So, |f^{(4)}(x)| = |-240x| = 240x.$$

On the interval from 5 *to* 8, $f^{(4)}(x)$ *maximizes when* $x = 8$

$$K = 240 * 8 = 1920$$

$$|E_s| \le \frac{K(b-a)^5}{180n^4}$$

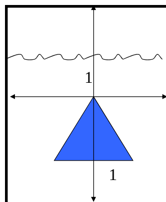
$$|E_s| \le \frac{1920(8-5)^5}{180(6)^4}$$

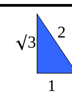
$$|E_s| \le 2$$

5

Compute the hydrostatic force felt on an equilateral triangle with sides of length 2 meters that is submerged pointed end up 1 meter below the water surface.

5





$$c^2 = a^2 + b^2 \rightarrow 2^2 = 1^2 + b^2 \rightarrow b = \sqrt{3}$$

widths:

The line $y = -\sqrt{3}x$ *fits the right side of the triangle.*

So, $x = \frac{y}{-\sqrt{3}}$ *and* $w_i = 2x = \frac{2y}{-\sqrt{3}}$

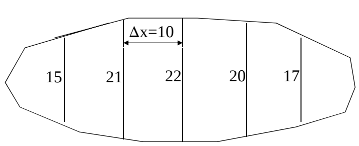
y_i	d_i
0	1
$-\sqrt{3}$	$\sqrt{3}+1$

$$F = \int_{-\sqrt{3}}^{0} \delta w_i d_i dy = \int_{-\sqrt{3}}^{0} 9800 \frac{2y}{-\sqrt{3}}(1-y)dy = 9800 \frac{2}{-\sqrt{3}} \int_{-\sqrt{3}}^{0} y - y^2 dy$$

$$= \frac{9800 \cdot 2}{-\sqrt{3}}\left[\frac{y^2}{2} - \frac{y^3}{2}\right]_{-\sqrt{3}}^{0} = \frac{9800*2}{-\sqrt{3}}\left[0 - \left(\frac{3}{2} + \frac{(\sqrt{3})^3}{3}\right)\right] = \frac{9800*2}{-\sqrt{3}}(-3.23) = 36,574.1N$$

5

A town wants to drain and fill a small, polluted swamp (see figure). The swamp averages a depth of 5 ft. Use Simpson's Rule to estimate the number of cubic yards of dirt it will take to fill the swamp after it is drained. (1 yd^3 = 27 ft^3)



Note: The height at the ends of the swamp is 0 ft.

5

$$Surface\ area = \frac{10}{3}(0 + 4(15) + 2(21) + 4(22) + 2(20) + 4(17) + 0)$$

$$Surface\ area = \frac{10}{3}(298) \approx 993.33\ ft^2$$

$$Volume = Surface\ area \times depth$$

$$Volume = 993.33 \times 5 = 4966.67\ ft^3$$

$$Volume = \frac{4966.67\ ft^3}{27\ ft^3} = 183.95\ yd^3$$

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